7

QUESTION PAPER SERIES CODE
A

Centre of Examination : ______________________

Roll No. : ______________________

Name of Candidate : ______________________

S A U

Entrance Test for M.Sc. (Applied Mathematics)

[2013]

Time : 3 hours *Maximum Marks* : 100

INSTRUCTIONS FOR CANDIDATES

Candidates must carefully read the following instructions before attempting the Question Paper :

(i) Write your Name and Roll Number in the space provided for the purpose on the top of this Question Paper and in the OMR/Answer Sheet.

(ii) This Question Paper has Three Parts : Part—A, Part—B and Part—C.

(iii) Part—A (Objective-type) has 20 questions of **1** mark each. All questions are compulsory.

(iv) Part—B (Objective-type) has 30 questions of **1** mark each. All questions are compulsory.

(v) Part—C (Objective-type) has 50 questions of **1** mark each. All questions are compulsory.

(vi) Symbols have their usual meanings.

(vii) **Please darken the appropriate Circle of 'Question Paper Series Code' on the OMR/Answer Sheet in the space provided.**

(viii) Questions for all the three parts should be answered on OMR/Answer Sheet.

(ix) Answers written by the candidates inside the Question Paper will **NOT** be evaluated.

(x) Calculators and Log Tables may be used.

(xi) Pages at the end have been provided for Rough Work.

(xii) **Return the Question Paper and the OMR/Answer Sheet** to the Invigilator at the end of the Entrance Test.

(xiii) **DO NOT FOLD THE OMR/ANSWER SHEET.**

INSTRUCTIONS FOR MARKING ANSWERS IN THE 'OMR SHEET'

1. **Please ensure that you have darkened the appropriate Circle of 'Question Paper Series Code' on the OMR Sheet in the space provided.**

2. Use only Blue/Black Ballpoint Pen to darken the Circle. Do not use Pencil to darken the Circle for Final Answer.

3. Please darken the whole Circle.

4. Darken ONLY ONE CIRCLE for each question as shown below in the example :

Example :

Wrong	Wrong	Wrong	Wrong	Correct
● ⓑ ⓒ ●	~~ⓐ~~ ⓑ ⓒ ⓓ	~~ⓐ~~ ⓑ ⓒ ~~ⓓ~~	⦿ ⓑ ⓒ ●	ⓐ ⓑ ⓒ ●

5. Once marked, no change in the answer is allowed.

6. Please do not make any stray marks on the OMR Sheet.

7. Please do not do any rough work on the OMR Sheet.

8. Mark your answer only in the appropriate circle against the number corresponding to the question.

9. There will be no negative marking in evaluation.

PART—A

1. The average age of a woman and her daughter is 16 years. The ratio of their ages is 7 : 1. Then the woman's age is

(a) 4 years

(b) 28 years

(c) 32 years

(d) 6 years

2. Tony is twelve years old. For three years, he has been asking his parents for a dog. His parents have told him that they believe a dog would not be happy in an apartment, but they have given him permission to have a bird. Tony has not yet decided what kind of bird he would like to have.

Find the statement that must be true according to the given information.

(a) Tony's parents like birds better than they like dogs

(b) Tony does not like birds

(c) Tony and his parents live in an apartment

(d) Tony and his parents would like to move

3. In an AP, the 6th term is half the 4th term and the 3rd term is 15. Then the 1st term and the common difference respectively are

(a) 21, 3

(b) 10, 15

(c) 21, −3

(d) −21, −3

4. If n is a prime number greater than 3, then the remainder when n^2 is divided by 12 is

(a) 0

(b) 1

(c) 2

(d) 3

5. A man shows his friend a woman sitting in a park and says that she is the daughter of my grandmother's only son. Then the relation between the two is

(a) sister-brother

(b) mother-son

(c) father-daughter

(d) husband-wife

6. To 15 litres of water containing 20% alcohol, we add 5 litres of pure water. Then the percentage of alcohol in water is

(a) 20

(b) 35

(c) 15

(d) 22·5

7. A train traveling at 36 km/hr crosses a platform in 20 s and a man standing on the platform in 10 s. Then the length of the platform in meters is

(a) 100

(b) 240

(c) 200

(d) 180

8. A 4 cm cube is cut into 1 cm cubes. Then the percentage increase in the surface area after such cutting is

(a) 300%

(b) 400%

(c) 200%

(d) 100%

9. Which one is grammatically correct option from the following?

(a) I have completed the work yesterday

(b) I completed the work yesterday

(c) I have had completed the work yesterday

(d) I has completed the work yesterday

10. The rice is being sold at \$27 per kg. During last month, its cost was \$24 per kg. Then by how much percent a family would reduce its consumption so as to keep the expenditure fixed?

(a) 10·2%

(b) 12·1%

(c) 11·1%

(d) None of the above

11. There are three persons P, Q and R having some balls each.

P gives Q and R as many balls as they already have
After some days, Q gives P and R as many balls as they have
After some days R gives P and Q as many balls as they have
Finally each has 24 balls

What are the original numbers of balls each had in the beginning?

(a) P-29, Q-21, R-12

(b) P-39, Q-21, R-12

(c) P-21, Q-12, R-29

(d) None of the above

12. Select the most appropriate meaning of the underlined idiomatic phrases :

Take care of what you say ! <u>You will have to eat your words !</u>

(a) You have no food to eat

(b) You will have to take back what you have said

(c) You are not good with your language

(d) None of the above

13. In an examination, out of 480 students, 85% of the girls and 70% of the boys passed. How many boys appeared in the examination if the total pass percentage was 75%?

(a) 370

(b) 320

(c) 360

(d) 380

14. Choose the sentence(s) where the underlined word is used correctly :

(i) This latest edition of novel is a <u>pedestrian</u> story about spies.

(ii) The exam paper is not <u>pedestrian</u> but difficult.

(iii) This is the <u>pedestrian</u> highway.

(iv) Every week we are forced to listen to a <u>pedestrian</u> lecture.

(a) (i) and (ii)

(b) (iii)

(c) (i) and (iv)

(d) None of the above

15. In the following sentence, choose the erroneous segment(s) :

He is one of those people (*P*)/who thinks (*Q*)/he owns the world (*R*).

(a) Error in *P*

(b) Errors in *P* and *Q*

(c) Errors in *Q* and *R*

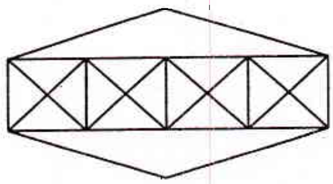
(d) None of the above

16. At a dinner party, every two guests used a bowl of rice between them, every three guests used a bowl of dal between them and every four used a bowl of meat between them. There were altogether 65 dishes. How many guests were present at the party?

(a) 60 (b) 61

(c) 63 (d) 55

17. Count the number of triangles and squares in the following figure :



(a) 30 triangles, 5 squares

(b) 40 triangles, 7 squares

(c) 33 triangles, 6 squares

(d) 31 triangles, 8 squares

18. Read the information given below and answer the question that follows :

Six friends are sitting in a circle and are facing the centre of the circle. L is between M and N. P is between Q and R. M and Q are opposite to each other.

Who are the neighbours of Q?

(a) M and L

(b) L and P

(c) P and N

(d) R and P

19. Various terms of an alphabet series are given with two terms missing as shown by (?) :

Z, S, W, O, T, K, Q, G, ?, ?

Choose the missing terms out of the alternatives given above.

(a) N, C

(b) N, D

(c) O, C

(d) O, D

20. What comes next in the following series?

3, 10, 101, ···

(a) 10101

(b) 10201

(c) 10202

(d) 11012

PART—B

21. If $f(x) = 2 + |x - 3|$ for all x, then the value of the derivative $f'(x)$ at $x = 3$ is

(a) -1

(b) 0

(c) 1

(d) Does not exist

22. If a function f is continuous for all x and if f has a relative maximum at $(-1, 4)$ and a relative minimum at $(3, -2)$, which of the following statements must be true?

(a) $f'(-1) = 0$

(b) The graph of f has a horizontal asymptote

(c) The graph of f has a horizontal tangent line at $x = 3$

(d) The graph of f intersects both the axes

23. $\lim\limits_{x \to 0+} \dfrac{e^{1/x} - 1}{e^{1/x} + 1}$ is equal to

(a) -1

(b) 1

(c) 0

(d) 2

24. A function $f(x)$ is defined as follows :

$$f(x) = \begin{cases} 1 + x & \text{if} \quad x \le 2 \\ 5 - x & \text{if} \quad x > 2 \end{cases}$$

Then

(a) $f(x)$ is continuous but not differentiable at $x = 2$

(b) $f(x)$ is differentiable at every point of R

(c) $f(x)$ is neither continuous nor differentiable at $x = 2$

(d) $f(x)$ is differentiable at $x = 2$ but is not continuous at $x = 2$

25. If $\frac{1}{u} = \sqrt{x^2 + y^2 + z^2}$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$ is equal to

(a) u

(b) $-u$

(c) 0

(d) ½

26. The derivative of $f(x) = \frac{x^4}{3} - \frac{x^5}{5}$ attains its maximum value at x equals to

(a) -1

(b) 0

(c) 1

(d) 4/3

27. The area of the region bounded by the lines $x = 0$, $x = 2$, $y = 0$ and the curve $y = e^{\frac{x}{2}}$ is

(a) $e - 1$

(b) $2(e - 1)$

(c) $2e - 1$

(d) $2e$

28. The degree of the differential equation

$$\left(\frac{d^3y}{dx^3}\right)^{\frac{2}{3}} = 1 + \frac{dy}{dx}$$

is

(a) 2

(b) 3

(c) 1

(d) 4

29. The differential equation for the family of curves $y = c_1 e^{2x} + c_2 e^{-2x}$, c_1 and c_2 are constants, is written as

(a) $\frac{d^2y}{dx^2} + 4y = 0$

(b) $\frac{d^2y}{dx^2} - 4y = 0$

(c) $\frac{d^2y}{dx^2} - 2y = 0$

(d) $\frac{d^2y}{dx^2} + 2y = 0$

30. The integrating factor for the linear differential equation

$$x \log x \frac{dy}{dx} + y = 2 \log x$$

is

(a) e^x

(b) $\log\left(\frac{1}{x}\right)$

(c) x^2

(d) $\log x$

31. The necessary and sufficient condition for the differential equation $A(x, y)\,dx + B(x, y)\,dy = 0$ to be exact is

(a) $\frac{\partial A}{\partial x} = -\frac{\partial B}{\partial y}$

(b) $\frac{\partial A}{\partial x} = \frac{\partial B}{\partial y}$

(c) $\frac{\partial A}{\partial y} = -\frac{\partial B}{\partial x}$

(d) $\frac{\partial A}{\partial y} = \frac{\partial B}{\partial x}$

32. If A and B are two square matrices of same order, then which of the following is not correct?

(a) $\det(A^T) = \det(A)$

(b) $\det(A^{-1}) = 1/\det(A)$

(c) $\det(AB) = \det(A)\det(B)$

(d) $\det(A+B) = \det(A) + \det(B)$

33. If A, B and C are three square matrices of same order $n \times n$, then which of the following is correct?

(a) rank $A = n$ and $AB = AC \Rightarrow B = C$

(b) rank $A = n$ and $AB = 0 \Rightarrow B \neq 0$

(c) $AB = 0$ but $A \neq 0$ and $B \neq 0 \Rightarrow$ rank $A = n$

(d) None of the above

34. The system of equations

$$\begin{aligned} 2x_1 + 2x_2 - 2x_3 &= 5 \\ 7x_1 + 7x_2 + x_3 &= 10 \\ 5x_1 + 5x_2 - x_3 &= 5 \end{aligned}$$

has

(a) no solution

(b) infinite solution

(c) one solution

(d) None of the above

35. Suppose A is a square matrix such that $A^2 = I$, the identity matrix of proper order, then

(a) $\det(A) = 0$

(b) $\det(A) = 2$

(c) $\det(A) = \pm 1$

(d) None of the above

36. The lines $2x - 3y = 5$ and $3x - 4y = 7$ are diameters of a circle of area 154 square units, then the equation of the circle is

(a) $x^2 + y^2 + 2x - 2y = 62$

(b) $x^2 + y^2 + 2x - 2y = 47$

(c) $x^2 + y^2 - 2x + 2y = 47$

(d) $x^2 + y^2 - 2x + 2y = 62$

37. The number of circles of a given radius which touch both the axes is

(a) 1

(b) 2

(c) 3

(d) 4

38. The parametric equations $x = 2 + t^2$, $y = 2t + 1$ represent

(a) a parabola with focus at (2, 1)

(b) a parabola with vertex at (2, 1)

(c) an ellipse with centre at (2, 1)

(d) a hyperbola with focus at (2, 1)

39. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 4\hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{c} = \hat{i} + \alpha\hat{j} + \beta\hat{k}$ are linearly dependent vectors and $|\vec{c}| = \sqrt{3}$, then

(a) $\alpha = 1, \beta = -1$

(b) $\alpha = 1, \beta = \pm 1$

(c) $\alpha = -1, \beta = \pm 1$

(d) $\alpha = \pm 1, \beta = 1$

40. If $\hat{a}$ and $\hat{b}$ are unit vectors such that $\hat{a}-4\hat{b}$ is at right angle to $7\hat{a}-2\hat{b}$, then the angle between $\hat{a}$ and $\hat{b}$ is

(a) $\frac{\pi}{2}$

(b) $\frac{\pi}{3}$

(c) $\frac{\pi}{6}$

(d) $\frac{\pi}{4}$

41. For $x \in [-1, 1]$, the value of $\cos^{-1} x + \cos^{-1}(-x)$ is

(a) 0

(b) π

(c) $2\cos^{-1} x$

(d) $\pi/2$

42. If $\cot x = -5/12$ for $x \in \left(\frac{\pi}{2}, \pi\right)$, then $\sec x$ equals to

(a) $5/12$

(b) $13/5$

(c) $-13/5$

(d) $-5/13$

43. If $\tan^{-1} a + \tan^{-1} b + \tan^{-1} c = \pi$, then

(a) $a+b+c = 2abc$

(b) $a+b+c = 0$

(c) $a+b+c = ab+bc+ca$

(d) $a+b+c = abc$

44. The number of solutions of the equation $\sin x + \cos x = 3$ is

(a) 2

(b) 1

(c) 0

(d) infinite

45. The probability that a man will hit a target is $\frac{2}{3}$. If he shoots at the target until he hits it for the first time, then the probability that it will take 5 shots to hit the target is

(a) $\frac{2}{243}$

(b) ${}^{5}C_{1}\,\frac{2}{243}$

(c) $\frac{2}{3}$

(d) $\frac{32}{243}$

46. A problem in Mathematics is given to three students A, B and C whose chances of solving it are $\frac{1}{2}, \frac{3}{4}$ and $\frac{1}{3}$ respectively. What is the probability that the problem will be solved if all of them try independently?

(a) $\frac{3}{24}$

(b) $\frac{6}{24}$

(c) $\frac{20}{24}$

(d) $\frac{22}{24}$

47. A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. The probability that it is actually a six is

(a) $\frac{3}{5}$

(b) $\frac{1}{5}$

(c) $\frac{3}{4}$

(d) None of the above

48. If $p(X) = \frac{2}{3}$, $p(Y) = \frac{1}{2}$ and $p(X \cup Y) = \frac{5}{6}$, then the events X and Y

(a) are mutually exclusive

(b) are independent as well as mutually exclusive

(c) are independent

(d) depend only on X

49. In an entrance examination, there are multiple choice questions. There are four possible answers to each question of which one is correct. The probability that a student knows the answer to a question is 90 percent. If the student gets the correct answer to the question, then the probability that the student was guessing is

(a) $\frac{1}{9}$

(b) $\frac{1}{37}$

(c) $\frac{36}{37}$

(d) $\frac{37}{40}$

50. The mode and median of observations 5, 4, 4, 3, 5, 3, 3, 4, 3, 5, 4, 3, 5 are

(a) mode = 3, median = 5

(b) mode = 3, median = 3

(c) mode = 4, median = 3

(d) mode = 3, median = 4

PART—C

51. If $f(x) = x^3 + x + 1$ and $g(x) = x \sin x$ are defined over $[0, 1]$, then which of the following is true?

(a) f is uniformly continuous and g is not uniformly continuous

(b) f is not uniformly continuous and g is uniformly continuous

(c) f and g are both uniformly continuous

(d) None of them is uniformly continuous

52. The equation $x^3 - 3x + 1 = 0$

(a) possesses no solution in $[0, 1]$

(b) has exactly one solution in $[0, 1]$

(c) has exactly two solutions in $[0, 1]$

(d) has all the three solutions in $[0, 1]$

53. If

$$f(x) = \begin{cases} x^3 \sin(\frac{1}{x}), & x \neq 0 \\ 0 \quad , & x = 0 \end{cases}$$

then

(a) $f(x)$ is not differentiable at $x = 0$

(b) $f(x)$ is differentiable at $x = 0$ but $f'(x)$ is not differentiable at $x = 0$

(c) $f'(x)$ is differentiable at $x = 0$ but $f''(x)$ is not differentiable at $x = 0$

(d) $f(x)$ is not continuous at $x = 0$

54. If $[x]$ denotes the greatest integer function, then the function $f(x) = [x \sin x]$ is differentiable in

(a) $(-1, 1)$

(b) $[-1, 1)$

(c) $(-1, 1]$

(d) $[-1, 1]$

55. If $f_n(x) = x^n$, $n = 1, 2, \cdots$ defined in $[0, 1]$, then which of the following satisfies?

(a) $f_n(x)$ is pointwise convergent but not uniformly convergent

(b) $f_n(x)$ is uniformly convergent

(c) $f_n(x)$ is not pointwise convergent

(d) $f_n(x)$ is pointwise convergent only at $x = 0$

56. The improper integral is defined by

$$\int_0^{\infty} \frac{\sin 2x}{x} dx$$

Then

(a) the integral is convergent and its value is $\pi/2$

(b) the integral is convergent and its value is π

(c) the integral is convergent and its value is 2π

(d) the integral is not convergent

57. In which interval, the infinite series

$$\sum_{n=0}^{\infty} x^n \cdot n!$$

is divergent?

(a) $(-\infty, 1)$

(b) $(-1, \infty)$

(c) $(-1, 1)$

(d) $(0, \infty)$

58. For the set $X = \{1, 2, 4, 7, 8, 11, 13, 14\}$ under multiplication modulo 15, which one is correct?

(a) $o(7) = 4$, $o(11) = 4$

(b) $o(7) = 1$, $o(11) = 4$

(c) $o(7) = 4$, $o(11) = 2$

(d) $o(7) = 2$, $o(11) = 4$

59. Let G be a group of non-zero real numbers under multiplication $K = \{x \in G : x \geq 1\}$ and $L = \{x \in G : x = 1 \text{ or } x \text{ is irrational}\}$. Then

(a) K and L are subgroups

(b) K is a subgroup but L is not

(c) L is a subgroup but K is not

(d) None of the above

60. Let H be a subgroup of G and g be an element of G. Then the set $K = \{ghg^{-1} : h \in H\}$

(a) is a normal subgroup of G

(b) is a subgroup of G

(c) is a cyclic subgroup of G

(d) None of the above

61. Let

$$W = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3 : x_1 - x_2 - x_3 = 0 \right\}$$

be the subset of $\mathbb{R}^3$. Then

(a) W is a vector subspace of $\mathbb{R}^3$

(b) W is not a vector subspace of $\mathbb{R}^3$

(c) W is a vector subspace of $\mathbb{R}^4$

(d) None of the above

62. Let

$$W = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3 : x_3 = x_1 - x_2 \right\}$$

be the subset of $\mathbb{R}^3$. Then

(a) W is spaned by the set $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \right\}$

(b) W is spaned by the set $\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \right\}$

(c) W is spaned by the set $\left\{ \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \right\}$

(d) W is spaned by the set $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix} \right\}$

63. The dimension of the solution space of the homogeneous linear system whose coefficient matrix $\begin{bmatrix} 1 & 1 & -1 & 1 \\ 0 & 1 & -2 & -1 \end{bmatrix}$ is

(a) 0

(b) 1

(c) 2

(d) 3

64. The general solution for the non-linear differential equation

$$\left(\frac{dy}{dx}\right)^2 - 5\frac{dy}{dx} + 6 = 0$$

is

(a) $(y - 3x + c)(y - 2x + c) = 0$

(b) $(y + 3x + c)(y - 2x + c) = 0$

(c) $(y - 3x + c)(y + 2x + c) = 0$

(d) $(y - 6x + c)(y - 5x + c) = 0$

65. Which of the following satisfies the differential equation

$$y\left(\frac{dy}{dx}\right)^2 + (x-y)\frac{dy}{dx} - x = 0?$$

(a) $(y+x+c)(y-x+c)=0$

(b) $(y+x+c)(y^2-x^2+c)=0$

(c) $(y-x+c)(y+x^2+c)=0$

(d) $(y-x+c)(y^2+x^2+c)=0$

66. The solution of the differential equation $\frac{dy}{dx} + x\sin(2y) = x^3\cos^2 y$ is

(a) $y = \frac{1}{2}(x^2-1) + ce^{-x^2}$

(b) $\sec y = \frac{1}{2}(x^2-1) + ce^{-x^2}$

(c) $y = \frac{1}{2}(\tan x - 1) + ce^{x}$

(d) $\tan y = \frac{1}{2}(x^2-1) + ce^{-x^2}$

67. The solution of the linear second-order differential equation

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 0$$

is

(a) $y = c_1e^{-2x} + c_2e^{x}$

(b) $y = c_1e^{-x} + c_2e^{x}$

(c) $y = c_1e^{-4x}$

(d) $y = e^{3x}$

68. The solution of linear partial differential equation

$$(mz - ny)\frac{\partial z}{\partial x} + (nx - lz)\frac{\partial z}{\partial y} = ly - mx$$

is

(a) $x + y + z = \psi(lx + my + nz)$

(b) $x^2 - y^2 + z^2 = \psi(lx + my + nz)$

(c) $x^2 + y^2 + z^2 = \psi(lx - my - nz)$

(d) $x^2 + y^2 + z^2 = \psi(lx + my + nz)$

69. The solution of non-linear partial differential equation

$$\left(\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2\right)y = z\frac{\partial z}{\partial y}, \quad z \equiv z(x, y)$$

is

(a) $z = (cx + d)^2 + c^2 y$

(b) $z^2 = cx + d + y^2$

(c) $z = (cx + d)^2 + c^2 y^4$

(d) $z^2 = (cx + d)^2 + c^2 y^2$

70. Which of the following represents the heat equation?

(a) $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$

(b) $\frac{\partial u}{\partial t} = -c^2 \frac{\partial^2 u}{\partial x^2}$

(c) $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial u}{\partial x}$

(d) $\frac{\partial^2 u}{\partial t^2} = -c^2 \frac{\partial^2 u}{\partial x^2}$

71. The Newton-Raphson formula for finding approximate root of $f(x) = 0$ is

(a) $x_{n+1} = \dfrac{f(x_n)}{f'(x_n)}$, $f'(x_n) \neq 0$

(b) $x_{n+1} = x_n + \dfrac{f(x_n)}{f'(x_n)}$, $f'(x_n) \neq 0$

(c) $x_{n+1} = x_{n-1} - \dfrac{f(x_n)}{f'(x_n)}$, $f'(x_n) \neq 0$

(d) $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$, $f'(x_n) \neq 0$

72. The forward difference $\Delta \tan^{-1} x$ can be evaluated as

(a) $\tan^{-1}\left(\dfrac{h}{x^2}\right)$

(b) $\tan^{-1}\left(\dfrac{h}{1+x^2}\right)$

(c) $\tan^{-1}\left(\dfrac{h}{x+h^2}\right)$

(d) $\tan^{-1}\left(\dfrac{h}{1+hx+x^2}\right)$

73. The highest order of polynomial integrand for which Simpson's $\frac{1}{3}$rd rule of integration is exact is

(a) 1

(b) 2

(c) 3

(d) 4

74. The secant formula for finding the square root of a real number R from the equation $x^2 - R = 0$ is

(a) $\dfrac{x_i x_{i-1} + R}{x_i + x_{i-1}}$

(b) $\dfrac{x_i x_{i-1}}{x_i + x_{i-1}} + R$

(c) $\dfrac{1}{2}\left(x_i + \dfrac{R}{x_i}\right)$

(d) $\dfrac{2x_i^2 + x_i x_{i-1} - R}{x_i + x_{i-1}}$

75. A square matrix $A = (a_{ij})_{n \times n}$ is lower triangular if

(a) $a_{ij} = 0,\ j > i$

(b) $a_{ij} = 0,\ i > j$

(c) $a_{ij} \neq 0,\ i > j$

(d) $a_{ij} \neq 0,\ j > i$

76. Given two vectors $\vec{a} = 2\hat{i} - 3\hat{j} + 6\hat{k}$ and $\vec{b} = -2\hat{i} + 2\hat{j} - \hat{k}$. The value of the term $\dfrac{\text{the projection of } \vec{a} \text{ on } \vec{b}}{\text{the projection of } \vec{b} \text{ on } \vec{a}}$ is

(a) $\dfrac{3}{7}$

(b) 7

(c) 3

(d) $\dfrac{7}{3}$

77. If $\vec{a}$ and $\vec{b}$ are any two unit vectors inclined to x-axis at 30° and 120°, then $|\vec{a} + \vec{b}|$ equals to

(a) $\sqrt{\dfrac{2}{3}}$

(b) $\sqrt{2}$

(c) $\sqrt{3}$

(d) 2

78. The shortest distance between the lines

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4} \text{ and } \frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}$$

is

(a) $\frac{1}{6}$

(b) $\frac{1}{\sqrt{6}}$

(c) $\frac{1}{\sqrt{3}}$

(d) $\frac{1}{3}$

79. The angle between the line $\vec{r} = (\hat{i} + 2\hat{j} - \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$ and the plane $\vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) = 4$ is

(a) $\sin^{-1}\left(\frac{\sqrt{2}}{3}\right)$

(b) $\sin^{-1}\left(\frac{1}{3}\right)$

(c) $\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$

(d) $\sin^{-1}\left(\frac{2}{3}\right)$

80. The value of $(\vec{b} \times \vec{c}) \times (\vec{c} \times \vec{a})$ equals to

(a) $[\vec{a}\,\vec{b}\,\vec{c}]\vec{a}$

(b) $[\vec{a}\,\vec{b}\,\vec{c}]\vec{b}$

(c) $[\vec{a}\,\vec{b}\,\vec{c}]\vec{c}$

(d) $[\vec{a}\,\vec{b}\,\vec{c}]$

81. The directional derivative of the function $f = x^2 - y^2 + 2z^2$ at the point $P(1, 2, 3)$ in the direction of the line PQ, where Q is the point $(5, 0, 4)$, is

(a) $\dfrac{28}{7\sqrt{3}}$

(b) $\dfrac{28}{3\sqrt{7}}$

(c) $\dfrac{3\sqrt{21}}{4}$

(d) $\dfrac{4\sqrt{21}}{3}$

82. The line integral $\oint_C (\phi \vec{\nabla} \psi \cdot d\vec{r})$ equals the surface integral

(a) $\iint_S [\vec{\nabla} \phi \times \vec{\nabla} \psi] \cdot \hat{n} ds$

(b) $\iint_S [\vec{\nabla} \psi \times \vec{\nabla} \phi] \cdot \hat{n} ds$

(c) $\iint_S [\vec{\nabla} \phi \cdot \vec{\nabla} \psi] \cdot \hat{n} ds$

(d) $\iint_S [\vec{\nabla} \phi \times \vec{\nabla} \psi] \times \hat{n} ds$

83. If $\vec{a} = (2\hat{i} + 3\hat{j} - \hat{k})$, $\vec{b} = (6\hat{i} + 9\hat{j} - 3\hat{k})$, then $\vec{a}$ and $\vec{b}$ are

(a) coplanar

(b) independent

(c) colinear

(d) All of the above

84. The value of p for which the vectors $(-\hat{i} + 5\hat{j} + p\hat{k})$ and $(p\hat{i} + 2\hat{j} + 3\hat{k})$ are perpendicular is

(a) 5

(b) 0

(c) −5

(d) 1

85. A ball is tossed downward from a point A with an initial speed of $V_A = 2 \cdot 0 \ \mathrm{m s^{-1}}$. If it takes one second to strike the ground, then the height is

(a) 2·9 meters

(b) 4·9 meters

(c) 6·9 meters

(d) 9·81 meters

86. A $2 \cdot 0 \times 10^3$ kg car travels at a constant speed of 12·0 m $\mathrm{s^{-1}}$ around a circular curve of radius 30·0 meters. As the car goes around the curve, the centripetal force is

(a) directed towards the center of the circular curve

(b) directed away from the center of the circular curve

(c) tangent to the curve in the direction of motion

(d) tangent to the curve opposite the direction of motion

87. The amount of work done against friction to slide a box in a straight line across a uniform horizontal floor depends most on the

(a) time taken to move the box

(b) distance the box is moved

(c) speed of the box

(d) direction of the box's motion

88. The set of vectors $\{(1, 1, 3, 1), (1, 2, 1, 1), (1, 0, 0, 1)\}$ is

(a) linearly independent

(b) linearly dependent

(c) neither linearly independent nor linearly dependent

(d) None of the above

89. The optimum value of the objective function of an LPP is attained at the

(a) points on x-axis

(b) points on y-axis

(c) points at the origin

(d) corner points of the feasible region

90. Which of the following sets is convex?

(a) $\{(x, y) : x^2 + y^2 \geq 1\}$

(b) $\{(x, y) : 2x^2 + 3y^2 \leq 6\}$

(c) $\{(x, y) : 4 \leq x^2 + y^2 \geq 9\}$

(d) None of the above

91. The LPP

Maximize $Z = 3x_1 + 2x_2$

subject to

$$x_1 - x_2 \leq 1$$
$$x_1 + x_2 \geq 3$$
$$x_1, x_2 \geq 0$$

has

(a) a unique solution

(b) infinitely many solutions

(c) an unbounded solution

(d) no solution

92. The convex hull of a set X is called a convex polyhedron if X consists of

(a) finite number of points

(b) infinite number of points

(c) no point

(d) Does not depend on number of points

93. The optimum value of the variables x_1, x_2 in LPP

Maximize $Z = 6x_1 + 4x_2$

subject to

$$2x_1 + 3x_2 \leq 30$$
$$3x_1 + 2x_2 \leq 24$$
$$x_1 + x_2 \geq 3$$
$$x_1, x_2 \geq 0$$

is

(a) $(8, 0)$

(b) $\left(\frac{12}{5}, \frac{42}{5}\right)$

(c) Both (a) and (b)

(d) None of the above

94. Three different numbers are selected at random from the set $X = \{1, 2, \cdots, 10\}$. The probability that the product of two of the numbers is equal to third is

(a) $\frac{3}{4}$

(b) $\frac{1}{40}$

(c) $\frac{1}{8}$

(d) $\frac{39}{40}$

95. A student obtained 60, 70 and 80 marks respectively in three monthly examinations in Mathematics and 95 in the final examination. The three monthly examinations are of equal weightage whereas the final examination is weighted twice as much as a monthly examination. The mean mark of Mathematics is

(a) 81

(b) 82

(c) 80

(d) 85

96. If mode of a data exceeds its mean by 12, then mode exceeds the median by

(a) 4

(b) 10

(c) 8

(d) 6

97. A man is dealt 4 spade cards from an ordinary pack of 52 cards. If he is given three more cards, then the probability that at least one of the additional card is also a spade is

(a) 0·47

(b) 0·23

(c) 0·64

(d) 0·77

98. Let R be a continuous random variable having probability density function

$$f(x) = \begin{cases} kx \ , & \text{if} \quad 0 \le x < 1 \\ k \ , & \text{if} \quad 1 \le x < 2 \\ -kx + 3k, & \text{if} \quad 2 \le x < 3 \\ 0 \ , & \text{elsewhere} \end{cases}$$

Then the value of k is

(a) $\frac{1}{3}$

(b) $\frac{1}{2}$

(c) $\frac{1}{6}$

(d) $\frac{1}{7}$

99. If variance of X is 1, then the variance of $2X + 3$ is

(a) 5

(b) 4

(c) 1

(d) 7

100. If for a discrete random variable X with a given probability density function

$$f(x) = \begin{cases} \frac{x}{15}, & \text{if} \quad x = 1, 2, 3, 4, 5 \\ 0, & \text{elsewhere} \end{cases}$$

then $p\left\{\frac{1}{2} < X < \frac{5}{2}\right\}$ for given $\{X > 1\}$ is

(a) $\frac{1}{7}$

(b) $\frac{2}{7}$

(c) $\frac{2}{15}$

(d) $\frac{1}{5}$

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