30005

QUESTION PAPER SERIES CODE
A

Test Centre : ______________________

Roll No. : ______________________

Name of the Candidate : ______________________

S A U

Entrance Test for M.Sc. (Applied Mathematics), 2016

[PROGRAMME CODE : MAM]

Question Paper

Time : 3 hours *Maximum Marks* : 100

INSTRUCTIONS FOR CANDIDATES

Please read carefully the following instructions before attempting the Question Paper :

(i) Write your Name, Roll Number and Name of the Test Centre in the space provided for the purpose on the top of this Question Paper and on the OMR Sheet.

(ii) This Question Paper has Two Parts : Part—A and Part—B.

(iii) Part—A (Objective-type) has 40 questions of **1** mark each. All questions are compulsory.

(iv) Part—B (Objective-type) has 60 questions of **1** mark each. All questions are compulsory.

(v) **A wrong answer will lead to the deduction of one-fourth ($\frac{1}{4}$) of the marks assigned to that question.**

(vi) Symbols have their usual meanings.

(vii) **Please darken the appropriate circle of 'Question Paper Series Code' and 'Programme Code' on the OMR Sheet in the space provided.**

(viii) All questions should be answered on the OMR Sheet.

(ix) Choose the one correct option out of the 4 options given for each question.

(x) Answers written inside the Question Paper will **NOT** be evaluated.

(xi) **Mobile Phones are NOT allowed.**

(xii) Pages at the end of the Question Paper have been provided for Rough Work.

(xiii) **Return the Question Paper and the OMR Sheet** to the Invigilator at the end of the Entrance Test.

(xiv) **DO NOT FOLD THE OMR SHEET.**

INSTRUCTIONS FOR MARKING ANSWERS ON THE 'OMR SHEET'

Use only BLUE/BLACK Ballpoint Pen

1. **Please ensure that you have darkened the appropriate circle of 'Question Paper Series Code' and 'Programme Code' on the OMR Sheet in the space provided.**

   Example :

   Question Paper Series Code

   Write Question Paper Series Code A or B in the box and darken the appropriate circle.

   ☐ A or B

   ●

   Ⓑ

   Programme Code

   Write Programme Code in the box and darken the appropriate circle.

   Write Programme Code ☐

MEC	○	MAM	●	PCS	○
MSO	○	MLS	○	PBT	○
MIR	○	PEC	○	PAM	○
MCS	○	PSO	○	PLS	○
MBT	○	PIR	○		

2. Use only a Blue or Black Ballpoint Pen to darken the circle. Do not use a pencil to darken the circle for the Final Answer.
3. Please darken the whole circle. ●
4. Darken <u>ONLY ONE CIRCLE</u> for each question as shown below in the example :

   Example :

Wrong	Wrong	Wrong	Wrong	Correct
● ⓑ ⓒ ●	~~ⓐ~~ ⓑ ⓒ ⓓ	~~ⓐ~~ ⓑ ⓒ ~~ⓓ~~	⊙ ⓑ ⓒ ●	ⓐ ⓑ ⓒ ●

5. Once marked, no change in the answer is possible.
6. Please do not make any stray marks on the OMR Sheet.
7. Please do not do any rough work on the OMR Sheet.
8. Mark your answer only in the appropriate circle against the number corresponding to the question.
9. **A wrong answer will lead to the deduction of one-fourth ($\frac{1}{4}$) of the marks assigned to that question.**
10. Write your six-digit Roll Number in the small boxes provided for the purpose; and also darken the appropriate circle corresponding to respective digits of your Roll Number as shown in the example below :

    Example :

ROLL NUMBER

1	3	5	7	2	0
●	①	①	①	①	①
②	②	②	②	●	②
③	●	③	③	③	③
④	④	④	④	④	④
⑤	⑤	●	⑤	⑤	⑤
⑥	⑥	⑥	⑥	⑥	⑥
⑦	⑦	⑦	●	⑦	⑦
⑧	⑧	⑧	⑧	⑧	⑧
⑨	⑨	⑨	⑨	⑨	⑨
⓪	⓪	⓪	⓪	⓪	●

PART—A

1. Which of the following sets is not open in $\mathbb{R}$?

   (a) The set $\mathbb{R}$ of real numbers

   (b) $(1, 3] \cup [2, 4)$

   (c) $\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots\right\}$

   (d) The empty set $\varnothing$

2. Consider the following statements:

   (i) Arbitrary union of open sets is open.

   (ii) Finite union of open sets is open.

   (iii) Arbitrary intersection of closed sets is closed.

   (iv) Finite intersection of closed sets is closed.

   Then

   (a) all the statements are correct

   (b) only (ii) and (iv) are correct

   (c) only (i), (ii) and (iv) are correct

   (d) only (ii), (iii) and (iv) are correct

3. If $f(x) = \sqrt{4-x}$ and $g(x) = x^2$, then the domain of the composition function $f \circ g$ is

   (a) $(-\infty, 4]$

   (b) $(-\infty, 2]$

   (c) $[-2, 2]$

   (d) $[-4, 4]$

4. $\lim_{x \to 0} \frac{|x|}{x}$ equals

   (a) 1

   (b) -1

   (c) 0

   (d) Does not exist

5. Consider $A = \sum_{n=0}^{\infty} a^n$, $a \in \mathbb{R}$. Then $A =$

(a) $\frac{1}{1-a}$

(b) $\frac{a}{1-a}$

(c) $\frac{1}{a-1}$

(d) does not exist always

6. Which of the following functions is uniformly continuous on $[0, \infty)$?

(a) $\sin x$

(b) $\sin x^2$

(c) $\sin \frac{1}{x}$, when $x \neq 0$ and $\sin 0 = 0$

(d) None of the above

7. The minimum value of the function $f(x, y) = y^2 + 4xy + 3x^2 + x^3$ is attained at the point

(a) $\left(-\frac{2}{3}, \frac{4}{3}\right)$

(b) $\left(\frac{2}{3}, -\frac{4}{3}\right)$

(c) $\left(\frac{1}{3}, \frac{4}{3}\right)$

(d) $\left(\frac{4}{3}, \frac{1}{3}\right)$

8. For the function $f(x, y) = e^{x^y}$, $\frac{\partial^2 f}{\partial x^2} =$

(a) $ye^{x^y}[yx^{y-1} + (y-1)x^{y-2}]$

(b) $ye^{x^y}[yx^{(y-1)^2} + (y-1)x^{y-2}]$

(c) $ye^{x^y}[yx^{2y-2} + (y-1)x^{y-2}]$

(d) $ye^{x^y}[yx^{2y-1} + (y-1)x^{y-2}]$

9. If $\vec{A} = x^2z^2\hat{i} - 2y^2z^2\hat{j} + xy^2z\hat{k}$, then the value of $\nabla \cdot \vec{A}$ at the point $(1, -1, 1)$ is

(a) -7

(b) -1

(c) 1

(d) 7

10. If $\{x_n\}$ and $\{y_n\}$ be two sequences such that $x_n + y_n \to 0$ as $n \to 0$, then

(a) both $\{x_n\}$ and $\{y_n\}$ must be convergent

(b) both $\{x_n\}$ and $\{y_n\}$ must be bounded

(c) at least one of $\{x_n\}$ and $\{y_n\}$ must be either convergent or bounded

(d) both $\{x_n\}$ and $\{y_n\}$ can be divergent

11. If $M = \begin{pmatrix} 1 & 2 & 2 \\ 0 & 2 & 2 \\ 0 & 1 & 1 \end{pmatrix}$ and $V = \{Mx^T : x \in R^3\}$, then $\dim V$ is

(a) 0

(b) 1

(c) 2

(d) 3

12. If $A^2 - A = O$, where A is a 9×9 matrix, then

(a) A must be a zero matrix

(b) A is an identity matrix

(c) rank of A is 1 or 0

(d) A is diagonalizable

13. If A is a unitary matrix, then eigenvalues of A are

(a) $1, -1$

(b) $1, -i$

(c) $-i, i$

(d) $-1, i$

14. If A is 5×5 matrix, all of whose entries are 1, then

(a) A is not diagonalizable

(b) A is idempotent

(c) A is nilpotent

(d) the minimal polynomial and the characteristic polynomial of A are not equal

15. The value of (1 2 3) (5 6 4 1) is

(a) (1 2 3 4 5 6)

(b) (1 2 4 3 6 5)

(c) (1 3 4 2 5 6)

(d) (1 2 3 5 6 4)

16. The order of 5 in the group {0, 1, 2, 3, 4, 5}, the composition being addition modulo 6, is

(a) 1

(b) 3

(c) 5

(d) 6

17. If a, b are any two elements of a group G, then $(ab)^2 = a^2b^2$ if and only if G is a/an

(a) group

(b) abelian group

(c) ring

(d) field

18. If f be an isomorphic mapping of a group G into a group G', then the order of an element a of G is equal to the order of

(a) $f(a-1)$

(b) identity

(c) $f(a)$

(d) 0

19. If p is a prime number and a is any integer, then a^p

(a) $\equiv a \pmod p$

(b) $\equiv p \pmod a$

(c) $\equiv 1 \pmod p$

(d) $\equiv 1 \pmod a$

20. To a permutation group, every finite group G is

(a) homomorphic

(b) isomorphic

(c) identical

(d) equal

21. The system of initial value problem

$$\frac{dx}{dt} = 3x + 8y, \ \frac{dy}{dt} = -x - 3y, \ x(0) = 6, \ y(0) = -2$$

possesses the solution

(a) $x = 4e^{t} + 2e^{-t}, \ y = -e^{t} - e^{-t}$

(b) $x = 4e^{t} + 200e^{-t}, \ y = e^{t} - e^{-t}$

(c) $x = e^{t} + 5e^{-t}, \ y = e^{t} - e^{-t}$

(d) $x = 4e^{5t} + 2e^{-t}, \ y = -e^{6t} - e^{-t}$

22. The solution of homogeneous PDE

$$\frac{\partial^3 u}{\partial x^3} - 2\frac{\partial^3 u}{\partial x^2 \partial y} - \frac{\partial^3 u}{\partial x \partial y^2} + 2\frac{\partial^3 u}{\partial y^3} = 0$$

is written as

(a) $\psi_1(y-x) + \psi_2(y^3 - x^3) + \psi_3(y+2x)$

(b) $\psi_1(2y+x) + \psi_2(y-9x) + \psi_3(8y+2x)$

(c) $\psi_1(y+3x) + \psi_2(y-2x) + \psi_3(5y+2x)$

(d) $\psi_1(y+x) + \psi_2(y-x) + \psi_3(y+2x)$

23. The Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$ in cylindrical polar coordinate is represented by

(a) $\frac{\partial^2 u}{\partial r^2} + \frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \phi^2} + \frac{\partial^2 u}{\partial z^2} = 0$

(b) $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \phi^2} + \frac{\partial^2 u}{\partial z^2} = 0$

(c) $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial \phi^2} + \frac{\partial^2 u}{\partial z^2} = 0$

(d) $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \phi^2} - \frac{\partial^2 u}{\partial z^2} = 0$

24. The slope at any point (x, y) of a curve is $1 + y/x$. If the curve passes through the point $(1, 1)$, then the equation of the curve is

(a) $y = x(1 + \ln(x))$

(b) $y = x(1 + \sin(x))$

(c) $y = x(1 + \cos(x))$

(d) $y = x(1 + \tan(x))$

25. The solution of the differential equation $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$, $y(0) = -1$, $y'(0) = 0$ is expressed as

(a) $e^{2x} - e^x$

(b) $e^{2x} - 2e^x$

(c) $e^{2x} - 2e^{-x}$

(d) $e^{8x} - 2e^{-x}$

26. The solution of the PDE $3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0$, $u(x, 0) = 4e^{-x}$ is

(a) $4\exp(2x - y)$

(b) $4\cosh\left(-\frac{1}{2}(2x - 3y)\right)$

(c) $4\sin\left(-\frac{1}{2}(2x - 3y)\right)$

(d) $4\exp\left(-\frac{1}{2}(2x - 3y)\right)$

27. The basis of the solution space to the differential equation

$$x^2\frac{d^2u}{dx^2} + x\frac{du}{dx} - u = 0, \quad (x \neq 0)$$

is

(a) $\left\{x, \frac{1}{x}\right\}$

(b) $\left\{1, \frac{1}{x}\right\}$

(c) $\{1, x\}$

(d) $\{1, x, x^2\}$

28. The value of β so that the differential equation $(x^3 + y)\,dx + (\beta x + y^3)\,dy = 0$ is exact

(a) 1

(b) 2

(c) −1

(d) 3

29. The most accurate value of $f(x) = x(\sqrt{x+1} - \sqrt{x})$ to six significant digits at $x = 500$ is

(a) 0

(b) 11·1500

(c) 11·1648

(d) 11·1748

30. Using Newton's method, the value of $\sqrt{5}$ at 1st iteration using starting value $x_0 = 2$ is

(a) 2·24

(b) 2·25

(c) 2·26

(d) 2·27

31. The coefficient of $(x+2)(x+1)$ in Newton's interpolating polynomial for the function satisfying the conditions $f(-2) = -15$, $f(-1) = -4$, $f(1) = 0$, $f(3) = 20$ is

(a) −3

(b) 3

(c) −4

(d) 0

32. The value of $f[-3, -2, 0, 4, 5]$ where $[,]$ denotes Newton's divided difference and $f(x) = 3x^4 - 4x^3 + 3x - 2$ is

(a) 0

(b) 1

(c) 2

(d) 3

33. Two numbers a and b are chosen at random from the set $\{1, 2, 3, \cdots, 3n\}$. The probability that $a^3 + b^3$ is divisible by 3, is

(a) $\frac{1}{2}$

(b) $\frac{1}{4}$

(c) $\frac{1}{3}$

(d) $\frac{1}{6}$

34. If A and B are two independent events such that $P(A^c \cap B) = \frac{2}{15}$ and $P(A \cap B^c) = \frac{1}{6}$, then $P(B)$ is

(a) $\frac{1}{5}$ or $\frac{4}{5}$

(b) $\frac{1}{6}$ or $\frac{4}{5}$

(c) $\frac{1}{6}$ or $\frac{5}{6}$

(d) $\frac{1}{5}$ or $\frac{5}{6}$

35. If $\frac{1-3p}{2}$, $\frac{1+4p}{3}$, $\frac{1+p}{6}$ are the probabilities of three mutually exclusive and exhaustive events, then the set of all values of p is

(a) $\left[-\frac{1}{4}, \frac{1}{3}\right]$

(b) $(0, 1)$

(c) $\left(0, \frac{1}{3}\right)$

(d) $(0, \infty)$

36. There are two persons A and B such that the chances of B telling the truth are twice that of A. A tells the truth in more than 25% cases. If A and B contradict each other in narrating the same statement, then the probability that A is telling the truth is

(a) $\frac{1}{4}$

(b) $\frac{1}{2}$

(c) $\frac{2}{3}$

(d) $\frac{3}{4}$

37. In the two-phase simplex method, the original problem may be a maximization or minimization problem but the phase–I problem

(a) is always a minimization problem

(b) is always a maximization problem

(c) may be minimization or maximization problem

(d) None of the above

38. If the leaving variable rule is not followed in some iteration of the simplex method, then the next table

(a) will not give a basic solution

(b) will give a basic solution which is not feasible

(c) will give a feasible solution

(d) None of the above

39. Which one of the following is an incorrect statement?

(a) All scarce resources have marginal profitability equal to zero.

(b) Shadow prices are also known as imputed values of the resources.

(c) A constraint $3x_1 - 7x_2 + 13x_3 - 4x_4 \geq -10$ can be equivalently written as $-3x_1 + 7x_2 - 13x_3 + 4x_4 \leq 10$.

(d) If all constraints of a minimization problem are '≥' type, then all dual variables are non-negative.

40. In linear programming, sensitivity analysis is a technique to

(a) allocate resources optimally

(b) minimize the cost of operations

(c) spell out the relation between primal and dual

(d) determine how optimal solution to LPP changes in response to problem inputs

PART—B

41. According to the fundamental theorem of calculus, if f is continuous on an interval $[a, b]$, then $\frac{d}{dx}\left(\int_a^x f(t)\,dt\right)$ equals

(a) $f(x)$

(b) $f(x) - f(a)$

(c) $f'(x) - f(a)$

(d) $f'(x) - f'(a)$

42. The point on the curve $y = x^2$, that is closest to the point (18, 0) is

(a) (4, 2)

(b) (2, 4)

(c) (0, 0)

(d) (3, 9)

43. For the sequence $\left\{-2, 2, -\frac{3}{2}, \frac{3}{2}, -\frac{4}{3}, \frac{4}{3}, \cdots\right\}$, the limit inferior and limit superior are respectively

(a) −2 and 2

(b) −1 and 1

(c) both are 0

(d) do not exist

44. If $\{a_n\}$ and $\{b_n\}$ be real sequences such that $a_n \leq b_n \ \forall n = 1, 2, \cdots$, then which of the following statements is true?

(a) If $\{a_n\}$ is convergent, then $\{b_n\}$ is also convergent.

(b) If $\{b_n\}$ is convergent, then $\{a_n\}$ is also convergent.

(c) If $\{b_n\}$ is divergent, then $\{a_n\}$ is also divergent.

(d) Nothing can be said.

45. The series $\sum_{n=1}^{\infty} x_n$ converges in the interval

(a) $(-1, 1)$

(b) $[-1, 1]$

(c) $(-\infty, \infty)$

(d) Diverges

46. $\lim_{x \to \infty} (e^x + x)^{1/x} =$

(a) e

(b) $e + 1$

(c) $\frac{1}{e}$

(d) Does not exist

47. If f is a Riemann integrable function, then which one of the following is true?

(a) $\int_a^b f'(x)\,dx = f(b) - f(a)$

(b) f is a continuous function

(c) $\int_a^b |f(x)|\,dx$ exists

(d) $|f|$ is a continuous function

48. $\int_0^{\pi/2} \log \sin x \, dx =$

(a) $\frac{\pi}{2} \log \frac{1}{2}$

(b) $\pi \log 2$

(c) $-\pi \log 2$

(d) $-\frac{\pi}{2} \log \frac{1}{2}$

49. Consider the function $f(x, y) = \frac{x - y}{(x + y)^3}$ and let

$$I_1 = \int_0^1 dx \int_0^1 f(x, y) \, dy \text{ and } I_2 = \int_0^1 dy \int_0^1 f(x, y) \, dx$$

Then

(a) $I_1 = \frac{1}{2}, \; I_2 = -\frac{1}{2}$

(b) $I_1 = -\frac{1}{2}, \; I_2 = \frac{1}{2}$

(c) $I_1 = I_2 = \frac{1}{2}$

(d) $I_1 = I_2 = -\frac{1}{2}$

50. Let $\{x_n\}$ be a sequence defined as follows :

$$x_1 = 0$$
$$8x_{n+1}^3 = 6x_n + 1, \quad n = 1, 2, 3, \cdots$$

Then the sequence $\{x_n\}$ is

(a) bounded and increasing

(b) only bounded

(c) only increasing

(d) neither bounded nor increasing

51. Let f and g be two functions such that $f(x) = g(x)$ for all $x \in \mathbb{Q}$, the set of rationals. Then $f(x) = g(x)$ for all $x \in \mathbb{R}$, if

(a) either f or g is continuous on $\mathbb{R}$

(b) f and g are both continuous on $\mathbb{Q}$

(c) f and g are both continuous on the complement of $\mathbb{Q}$

(d) f and g are both continuous on $\mathbb{R}$

52. Let $f : A \subset \mathbb{R} \to \mathbb{R}$ be a function such that $f'(x) \neq 0$ for all $x \in A$. Then the function f is

(a) bounded

(b) increasing

(c) decreasing

(d) one-one

53. Let f be Riemann integrable on $[a, b]$. Define

$$F(x) = \int_a^x f(t)\,dt, \quad \forall\, x \in [a, b]$$

Then which one of the following statements is not true?

(a) F is continuous.

(b) F is differentiable.

(c) If f is continuous, then F is continuous.

(d) If f is continuous, then F is differentiable.

54. If $\vec{A} = x^2z^2\hat{i} - 2y^2z^2\hat{j} + xy^2z\hat{k}$, then the value of $\nabla \times \vec{A}$ at the point $(1, -1, 1)$ is

(a) $2\hat{i} + \hat{j}$

(b) $2\hat{i} - \hat{j}$

(c) $\hat{i} + 2\hat{j}$

(d) $\hat{i} - 2\hat{j}$

55. If A is a 3×3 matrix over R, then $(t^2+1)(t^2+2)$

(a) is a minimal polynomial

(b) is a characteristic polynomial

(c) Both (a) and (b) are true

(d) Neither (a) nor (b) is true

56. If M is a 2-square matrix of rank 1, then M is

(a) diagonalizable and non-singular

(b) diagonalizable and nilpotent

(c) neither diagonalizable nor nilpotent

(d) either diagonalizable or nilpotent

57. If A is a 4-square matrix and $A^5 = O$, then

(a) $A^4 = I$

(b) $A^4 = A$

(c) $A^4 = O$

(d) $A^4 = -I$

58. If I be the identity transformation of the finite-dimensional vector space V, then the nullity of I is

(a) $\dim V$

(b) 0

(c) 1

(d) $\dim V - 1$

59. If A be $m \times n$ matrix of rank n with real entries, then which of the following statements is correct?

(a) $Ax = b$ has a solution for any b.

(b) $Ax = 0$ does not have a solution.

(c) If $Ax = b$ has a solution, then it is unique.

(d) $y'A = 0$ for some non-zero y, where y' denotes the transpose of vector y.

60. If the elements a, b of a group commute and $o(a) = m$, $o(b) = n$, where m and n are relatively prime, then $o(ab)$ is

(a) m

(b) n

(c) mn

(d) None of the above

61. Let H_1, H_2 be two distinct subgroups of a finite group G, each of order 2. Let H be the smallest subgroup containing H_1 and H_2. Then the order of H is

(a) always 2

(b) always 4

(c) always 8

(d) None of the above

62. Every homomorphic image of a group G is isomorphic to some ____ groups of G.

(a) Abelian

(b) normal

(c) quotient

(d) permutation

63. If S be the collection of (isomorphism classes of) groups G which have the property that every element of G commutes only with the identity element and itself, then

(a) $|S| = 1$

(b) $|S| = 2$

(c) $|S| \geq 3$ and is finite

(d) $|S| = \infty$

64. For a group G, if $F(G)$ denotes the collection of all subgroups of G, which of the following situations can occur?

(a) G is finite but $F(G)$ is infinite

(b) G is infinite but $F(G)$ is finite

(c) G is countable but $F(G)$ is uncountable

(d) G is uncountable but $F(G)$ is countable

65. A polynomial of odd degree with real coefficients must have

(a) at least one real root

(b) no real root

(c) only real roots

(d) at least one root which is not real

66. Which of the following primes satisfies the congruence $a^{24} \equiv 6a + 2 \bmod 13$?

(a) 41

(b) 47

(c) 67

(d) 83

67. The algebraic structure $(\{0, 1, 2, 3\}, +_4, *_4)$ is a/an

(a) ring

(b) integral domain

(c) field

(d) skew field

68. A finite commutative ring with 0 is a/an

(a) ring

(b) integral domain

(c) field

(d) skew field

69. The differential equation $\left|\frac{du}{dx}\right| + |u| = 0$ has

(a) no solution

(b) one non-zero solution

(c) only trivial solution

(d) infinitely many non-zero solutions

70. The differential equation $\left|\frac{du}{dx}\right| + |u| + 5 = 0$ has

(a) no solution

(b) one non-zero solution

(c) only trivial solution

(d) infinitely many non-zero solutions

71. If I, R and L denote current, resistance and inductance in an electrical circuit and satisfy the initial value problem

$$L\frac{dI}{dt} + RI = 0, \; I(0) = 5$$

then the value of the current at time t is given by

(a) $5e^{-\left(\frac{R}{L}\right)t}$

(b) $7e^{-\left(\frac{R}{L}\right)t}$

(c) $8e^{-\left(\frac{R}{L}\right)t}$

(d) $5e^{\left(\frac{R}{L}\right)t}$

72. The general solution of the linear non-homogeneous partial differential equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial y} - 6\frac{\partial^2 u}{\partial y^2} = y\cos(x)$$

is

(a) $\varphi_1(y - x) + \varphi_2(y + x) + \sin(x) - y\cos(5x)$

(b) $\varphi_1(4y - 3x) + \varphi_2(3y + 2x) + \sin(5x) - y\cos(x)$

(c) $\varphi_1(y - 3x) + \varphi_2(y + 2x) + \sin(x) - y\cos(x)$

(d) $\varphi_1(y - 3x) + \varphi_2(3y + 2x) + \sin(2x) - y\cos(x)$

73. The following partial differential equation

$$\frac{\partial^2 u}{\partial x^2} + 3\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = 0$$

is classified as

(a) elliptic

(b) non-linear

(c) parabolic

(d) hyperbolic

74. Where, in the xy-plane, is the differential equation

$$\frac{dy}{dx} = \frac{1+x^2}{3y-y^2}$$

guaranteed to have a unique solution?

(a) Everywhere except for $y \neq 0$

(b) Everywhere except for $y \neq 3$

(c) Everywhere except for $y \neq 0$ and $y \neq 3$

(d) The solution does not exist anywhere

75. The general solution to the ordinary differential equation

$$\frac{d^2u}{dx^2} + 6\frac{du}{dx} + 9u = 0$$

is $u(x) = Axe^{-3x} + Be^{-3x}$. Which of the following options is correct?

(a) As $x \to \infty$, $u \to A$ for any value of B

(b) The behaviour of u as $x \to \infty$ depends on A and B

(c) As $x \to \infty$, $u \to \infty$ for any value of A and B

(d) As $x \to \infty$, $u \to 0$ for any value of A and B

76. The partial differential equation that governs the family of surfaces $u(x, y) = (x-\alpha)^2 + (y-\beta)^2$ is given by

(a) $\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = 4u$

(b) $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 4u$

(c) $\left(\frac{\partial u}{\partial x}\right)^4 + \left(\frac{\partial u}{\partial y}\right)^4 = 4u$

(d) $\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = u$

77. Which one of the following represents a one-space dimensional wave equation in Cartesian coordinates system?

(a) $\frac{\partial u}{\partial x} = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2}$

(b) $\frac{\partial^4 u}{\partial x^4} = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2}$

(c) $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2}$

(d) $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2} \frac{\partial^4 u}{\partial t^4}$

78. Consider the differential equation

$$\frac{d^2 u}{dx^2} + 2\frac{du}{dx} - 8u = 0$$

The values of B for which the given differential equation has solution of the form $u(x) = e^{Bx}$ are

(a) 2, 4

(b) 2, −4

(c) 4, −4

(d) 1, −4

79. The characteristic curve of $2y\frac{\partial u}{\partial x} + (2x + y^2)\frac{\partial u}{\partial y} = 0$ passing through (0, 0) is given by

(a) $u^4 = 2(e^x - x - 1)$

(b) $u^2 = 2(e^x - x - 1)$

(c) $u^2 = 2(e^x + 5x + 1)$

(d) $u = \sin(x) - x - 1$

80. Using four digits rounding arithmetic, the most accurate value of the smaller root of the quadratic equation $0 \cdot 2x^2 - 47 \cdot 91x + 6 = 0$ is

(a) 0·1132

(b) 0·1200

(c) 0·1253

(d) 0·1500

81. For the function $f(x) = x^3 + x^2 - 3x - 3$, the iteration function

$$g(x) = \sqrt{(3 + 3x - x^2)/x}$$

converges to zero α of f with initial guess $x_0 = 1 \cdot 0$. Then α is

(a) -1

(b) $-\sqrt{3}$

(c) $\sqrt{3}$

(d) 1

82. The order of convergence of the fixed point iteration scheme for the iteration function

$$g(x) = \frac{1}{2}\left(x + \frac{a}{x}\right)$$

to the fixed point is

(a) 1

(b) 2

(c) $\sqrt{2}$

(d) 3

83. For the matrix $A = \begin{pmatrix} 1 & -2 \\ 4 & 3 \end{pmatrix}$, the l_∞ norm is

(a) 3

(b) 7

(c) 4

(d) 1

84. The Gauss-Seidel method with initial guess $x^{(0)} = [0, 0, 0]^T$ applied to the system of equations

$$\begin{aligned} 5x_1 + x_2 + 2x_3 &= 10 \\ -3x_1 + 9x_2 + 4x_3 &= -14 \\ x_1 + 2x_2 - 7x_3 &= -33 \end{aligned}$$

gives the solution at 1st iteration as

(a) $x^{(1)} = [2\cdot00, -0\cdot89, 4\cdot75]^T$

(b) $x^{(1)} = [2\cdot00, 0\cdot89, 4\cdot75]^T$

(c) $x^{(1)} = [1\cdot00, -0\cdot89, 4\cdot75]^T$

(d) $x^{(1)} = [2\cdot00, 0\cdot89, -4\cdot75]^T$

85. Newton's method applied to the system of non-linear algebraic equations with initial guess $x^{(0)} = [1, 1, 1]^T$

$$\begin{aligned} x_1^3 - 2x_2 - 2 &= 0 \\ x_1^3 - 5x_3^2 + 7 &= 0 \\ x_2 x_3^2 - 1 &= 0 \end{aligned}$$

gives the approximate solution at 1st iteration as

(a) $x^{(1)} = [1\cdot43, 0\cdot143, 1\cdot43]^T$

(b) $x^{(1)} = [1\cdot43, 0, 1\cdot43]^T$

(c) $x^{(1)} = [1\cdot00, 0\cdot143, 1\cdot43]^T$

(d) $x^{(1)} = [1\cdot43, 0\cdot143, 1\cdot00]^T$

86. The coefficient of $x^2(x-2)$ in Newton's form of Hermite interpolating polynomial of the function $f(0) = 0$, $f(2) = 2e^{-2}$, $f(4) = 4e^{-4}$, $f'(0) = 1$, $f'(2) = -e^{-2}$, $f'(4) = -3e^{-4}$ is

(a) $\dfrac{2e^{-2} - 1}{2}$

(b) $\dfrac{-2e^{-2} - 1}{2}$

(c) $\dfrac{3e^{-2} - 1}{2}$

(d) $\dfrac{e^{-2} - 1}{2}$

87. If $x \in [0, 5]$, then what is the probability that $x^2 - 3x + 2 \geq 0$?

(a) $\frac{4}{5}$

(b) $\frac{1}{5}$

(c) $\frac{2}{5}$

(d) $\frac{3}{5}$

88. The probability of India winning a test match against Australia is $\frac{2}{3}$. Assuming independence from match to match, the probability that in a 7-match series India's third win occurs at the fifth match, is

(a) $\frac{1}{4}$

(b) $\frac{3}{4}$

(c) $\frac{2}{3}$

(d) $\frac{1}{2}$

89. Let X be a random variable having Poisson distribution. If the probability of success $p = 0 \cdot 001$, then find out the number of trials required for attaining at least one success with more than 99% surety, i.e., $P(X \geq 1) \geq 0 \cdot 99$ is

(a) $n \geq 4600$

(b) $n \geq 4300$

(c) $n \geq 2300$

(d) $n \geq 2700$

90. If the two lines of regression, viz., y on x and x on y are $y = \frac{1}{7}x - 11$, $x = cy + 12$ respectively, then the limit within which the constant c must lie is given by

(a) $-1 \leq c \leq 1$

(b) $5 \leq c \leq 6$

(c) $0 \leq c \leq 7$

(d) $3 \leq c \leq 5$

91. A continuous random variable X has a probability density function

$$f(x) = \begin{cases} x^2, & 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$$

If $P(X \le a) = P(X > a)$, then

(a) $a = \left(\frac{1}{2}\right)^{\frac{1}{3}}$

(b) $a = \frac{1}{\sqrt{2}}$

(c) $a = \frac{1}{2}$

(d) None of the above

92. A random variable X has the following probability distribution :

X	1	2	3	4	5	6	7	8
$P(X)$	0·15	0·23	0·12	0·10	0·20	0·08	0·07	0·05

For the events $E = \{X \text{ is a prime number}\}$, $F = \{X < 4\}$, the probability $P(E \cup F)$, is

(a) 0·50

(b) 0·77

(c) 0·35

(d) 0·87

93. A rifleman is firing at a distant target and has only a 10% chance of hitting it. The least number of rounds he must fire in order to have more than 50% chance of hitting it at least once is

(a) 11

(b) 5

(c) 9

(d) 7

94. The following five inequalities define a feasible region. Which one of these could be removed from the list without changing the region?

(a) $x - 2y \geq -8$

(b) $x \geq 0;\ y \geq 0$

(c) $-x + y \leq 10$

(d) $x + y \leq 20$

95. A basic solution is called degenerate if

(a) the value of at least one of the basic variables is non-zero

(b) the value of all the basic variables is zero

(c) the value of at least one of the basic variables is zero

(d) the value of all the basic variables is non-zero

96. Worstcase complexity of simplex method as formulated by Dantzig is

(a) exponential time

(b) polynomial time

(c) Cannot be determined

(d) None of the above

97. The situation in which objective function is parallel to the binding constraint of direction optimization is classified as

(a) negative optimal solution

(b) positive optimal solution

(c) alternative optimal solution

(d) regular optimal solution

98. The analysis which is used to determine the effect of coefficient change with the same current basis is classified as

(a) parameter analysis

(b) original analysis

(c) sensitivity analysis

(d) formulation analysis

99. For every (=) to constraint, the variable which is added to left side of the equation is classified as

(a) original variable

(b) artificial variable

(c) additive variable

(d) non-additive variable

100. The non-negative variable subtracted from any less than or equal to constraint of simplex method is classified as

(a) surplus variable

(b) deficit variable

(c) right variable

(d) left constant

★★★

SPACE FOR ROUGH WORK