Test Centre : ____________________

Roll No. : ____________________

Name of the Candidate : ____________________

# S A U

## Entrance Test for M.Sc. (Applied Mathematics) 2017

**[ PROGRAMME CODE : 30005 ]**

**Question Paper Series Code : A**

## QUESTION PAPER

*Time* : 3 hours *Maximum Marks* : 100

**INSTRUCTIONS FOR CANDIDATES**

*Read carefully the following instructions before attempting the Question Paper* :

(i) Write your Name, Roll Number and Name of the Test Centre in the space provided for the purpose on the top of this Question Paper and on the OMR Sheet.

(ii) This Question Paper has Two Parts : Part—A and Part—B.

(iii) Part—A (objective-type) has 40 questions of **1** mark each. All questions are compulsory. Part—B (objective-type) has 60 questions of **1** mark each. All questions are compulsory.

(iv) **A wrong answer will lead to the deduction of one-fourth ($\frac{1}{4}$) of the marks assigned to that question.**

(v) Symbols have their usual meanings.

(vi) **Please darken the appropriate circle of 'Question Paper Series Code' and 'Programme Code' on the OMR Sheet in the space provided.**

(vii) All questions should be answered on the OMR Sheet.

(viii) Answers written inside the Question Paper will **NOT** be evaluated.

(ix) **Non-programmable calculators and log tables may be used. Mobile phones are NOT allowed.**

(x) Pages at the end of the Question Paper have been provided for Rough Work.

(xi) **Return the Question Paper and the OMR Sheet** to the Invigilator at the end of the Entrance Test.

(xii) **DO NOT FOLD THE OMR SHEET.**

## INSTRUCTIONS FOR MARKING ANSWERS ON THE 'OMR SHEET'

**Use BLUE/BLACK Ballpoint Pen Only**

1. **Please ensure that you have darkened the appropriate circle of 'Question Paper Series Code' and 'Programme Code' on the OMR Sheet in the space provided.**

   **Question Paper Series Code**

   Write Question Paper Series Code **A** or **B** in the box and darken the appropriate circle.

   [ ] **A** or **B**

   ●

   Ⓑ

2. Use only Blue/Black Ballpoint Pen to darken the circle. Do not use Pencil to darken the circle for Final Answer.
3. Please darken the whole circle. ●
4. Darken <u>ONLY ONE CIRCLE</u> for each question as shown below in the example :

***Example*** **:**

| Wrong | Wrong | Wrong | Wrong | Correct |
|---|---|---|---|---|
| ● ⓑ ⓒ ● | ⓐ̸ ⓑ ⓒ ⓓ | ⓐ̸ ⓑ ⓒ ⓓ̸ | ⊙ ⓑ ⓒ ● | ⓐ ⓑ ⓒ ● |

5. Once marked, no change in the answer is possible.
6. Please do not make any stray marks on the OMR Sheet.
7. Please do not do any rough work on the OMR Sheet.
8. Mark your answer only in the appropriate circle against the number corresponding to the question.
9. **A wrong answer will lead to the deduction of one-fourth of the marks assigned to that question.**
10. Write your six-digit Roll Number in small boxes provided for the purpose; and also darken the appropriate circle corresponding to respective digits of your Roll Number as shown in the example below.

***Example*** **:**

**ROLL NUMBER**

| 1 | 3 | 5 | 7 | 2 | 0 | 2 |
|---|---|---|---|---|---|---|
| ● | ① | ① | ① | ① | ① | ① |
| ② | ② | ② | ② | ● | ② | ● |
| ③ | ● | ③ | ③ | ③ | ③ | ③ |
| ④ | ④ | ④ | ④ | ④ | ④ | ④ |
| ⑤ | ⑤ | ● | ⑤ | ⑤ | ⑤ | ⑤ |
| ⑥ | ⑥ | ⑥ | ⑥ | ⑥ | ⑥ | ⑥ |
| ⑦ | ⑦ | ⑦ | ● | ⑦ | ⑦ | ⑦ |
| ⑧ | ⑧ | ⑧ | ⑧ | ⑧ | ⑧ | ⑧ |
| ⑨ | ⑨ | ⑨ | ⑨ | ⑨ | ⑨ | ⑨ |
| ⓪ | ⓪ | ⓪ | ⓪ | ⓪ | ● | ⓪ |

## PART—A

1. If $f(x) = \sin|x|$, $x \in [-2\pi, 2\pi]$, then

   a. $f(x) \geq 0$ for all $x \in [-2\pi, 2\pi]$

   b. $f(x) < 0$ for $x \in [-2\pi, 0)$ and $f(x) > 0$ for $x \in [0, 2\pi]$

   c. $f$ is differentiable on $[-2\pi, 2\pi]$

   d. $f$ is continuous but not differentiable at $x = 0$

2. If $[\cdot]$ denotes the greatest integer function, then

$$\int_0^{1\cdot 5} [x^2]\, dx =$$

   a. 2·25

   b. 9/8

   c. $2 - \sqrt{2}$

   d. 1/2

3. If $x^2 + y^2 = 1$, $x, y \in \mathbb{R}$, then the bounds of $x + y$ are

   a. $0, \sqrt{2}$

   b. $-\sqrt{2}, \sqrt{2}$

   c. $-1, 1$

   d. $0, 2$

4. If $y = \cos x - 1$, $x \in [0, 2\pi]$, then the point(s) on this curve at which the tangent is parallel to the $x$-axis is/are

   a. $(0, 0)$, $(2\pi, 0)$, $(\pi, -2)$

   b. $(\pi, 2)$

   c. $(2\pi, 0)$, $(\pi, -2)$

   d. $(\pi, 3)$, $(\pi, 1)$

5. The value of

$$\lim_{n \to \infty} \frac{1+2^{\frac{1}{2}}+3^{\frac{1}{3}}+4^{\frac{1}{4}}+\cdots+n^{\frac{1}{n}}}{n}$$

is

a. 0

b. 1

c. $\infty$

d. not defined

6. The roots of the equation $x = e^{-x}$ lie in

a. I quadrant

b. II quadrant

c. III quadrant

d. IV quadrant

7. The domain of the function

$$f(x) = \frac{\sin^{-1} x}{x}$$

is

a. $[-1, 0) \cup \{1\}$

b. $(0, 1]$

c. $\mathbb{R} \setminus \{0\}$

d. $[-1, 0) \cup (0, 1]$

8. The minimum value of

$$x + \frac{1}{x}, x > 0$$

is

a. 3/2

b. 1

c. 2

d. Cannot be determined

9. Which one of the following statements is correct?

a. Every bounded sequence is convergent.

b. Every bounded sequence has a convergent subsequence.

c. Every bounded sequence having a limit point is convergent.

d. A convergent sequence may have a divergent subsequence.

10. $\lim_{x \to 0} \frac{e^{\frac{1}{x}}}{e^{\frac{1}{x}+1}} =$

a. 0

b. 1

c. Does not exist

d. None of the above

11. The determinant of the matrix

$$\begin{pmatrix} 1 & z & -y \\ -z & 1 & x \\ y & -x & 1 \end{pmatrix}$$

is

a. $1+x+y+z$

b. $1+xyz$

c. $1+x^2+y^2+z^2$

d. None of the above

12. If $A$ is a $n*1$ non-zero matrix and $B$ is $1*n$ non-zero matrix, then

a. Rank $(AB) = 1$

b. Rank $(AB) = n$

c. Rank $(AB) = 0$

d. None of the above

13. Let $A$ be the matrix of order $m*n$. Then the determinant of $A$ exists if and only if

a. $m > n$

b. $m$ equals to $n$

c. $m$ not equals to $n$

d. $m < n$

14. If $T: R^4 \to R^4$, defined by $T(e_1) = e_2$, $T(e_2) = e_3$, $T(e_3) = 0$, $T(e_4) = e_3$, then

a. $T$ is nilpotent

b. $T$ has non-zero eigenvalue

c. index of nilpotent is five

d. $T$ is not nilpotent

15. Let $a = (123)(145)$. Then $a^4$ is equal to

a. $a$

b. $a^2$

c. $a^{-1}$

d. $a^{-2}$

16. A commutative division ring is

a. a vector space

b. a group

c. a finite integral domain

d. a field

17. If $(G, *)$ is a group and $\forall\, a, b \in G$, $b^{-1}*a^{-1}*b*a = e$, then $G$ is

a. an Abelian group

b. a non-Abelian group

c. a ring

d. a field

18. The number of elements in the conjugacy class of the 3-cycle (2 3 4) in the symmetric group $S_6$ is

a. 20

b. 40

c. 120

d. 216

19. If $G$ is a group such that $(ab)^2 = a^2b^2 \forall a, b \in G$, then $G$ is

a. a finite group

b. a noncyclic group

c. an Abelian group

d. a non-Abelian group

20. Which one of the following statements is correct?

a. Cyclic groups may have noncyclic subgroups

b. Noncyclic groups have no cyclic subgroup

c. Abelian groups may have non-Abelian subgroups

d. Non-Abelian groups may have Abelian subgroups

21. The integrating factor for the Leibnitz linear equation

$$\frac{dy}{dx} + P(x)y = Q(x)$$

is

a. $e^{\int P(x)\,dx}$

b. $e^{\int Q(x)\,dx}$

c. $\int P(x)\,dx$

d. $\int Q(x)\,dx$

**22.** The solution of nonlinear differential equation

$$x^2\left(\frac{dy}{dx}\right)^2 + xy\frac{dy}{dx} - 6y^2 = 0$$

is

a. $(y - cx^5)(yx^3 - c) = 0$

b. $(y - cx^2)(yx^3 - c) = 0$

c. $(y - cx^2)(yx^7 - c) = 0$

d. $(y - cx^6)(yx^3 - c) = 0$

**23.** The equation $e^x dx + e^y dy = 0$ is of the order

a. infinite

b. zero

c. one

d. three

**24.** The general solution of the differential equation

$$\frac{dy}{dx} = -\frac{x}{y}$$

is

a. $x^4 + y^4 = 2c$

b. $x^4 + y^2 = 2c$

c. $x^2 + y^4 = 2c$

d. $x^2 + y^2 = 2c$

**25.** The solution of an ordinary differential equation of order $n$ contains

a. two constants

b. exactly one constant

c. $n$ arbitrary constants

d. more than $n$ arbitrary constants

26. For nonhomogeneous equation

$$\frac{dy}{dx} + P(x)y = Q(x)$$

if $y_1(x)$ and $y_2(x)$ are its solutions, then the solution of the corresponding homogeneous equation

$$\frac{dy}{dx} + P(x)y = 0$$

is

a. $y(x) = y_1(x) - y_2(x)$

b. $y(x) = y_1(x) + y_2(x)$

c. $y(x) = y_1(x)y_2(x)$

d. $y(x) = y_1(x) / y_2(x)$

27. The complete solution of a partial differential equation

$$2\frac{\partial^2 z}{\partial x^2} + 5\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = 0$$

is

a. $z(x, y) = f(y - x) + g(y - \frac{x}{2})$

b. $z(x, y) = f(y - 2x) + g(y - \frac{x}{2})$

c. $z(x, y) = f(y + 2x) + g(y - 7x)$

d. $z(x, y) = f(y - 2x) + g(y - x)$

28. Truncation error is caused by approximating

a. irrational numbers

b. fractions

c. rational numbers

d. exact mathematical procedures

**29.** The expression for true error in calculating the derivative of $\sin(2x)$ at $x = \frac{\pi}{4}$ by using the approximate expression

$$\frac{df}{dx} = \frac{f(x+h) - f(x)}{h}$$

is

a. $\frac{h - \cos(h) - 1}{h}$

b. $\frac{1 - \cos(2h)}{h}$

c. $\frac{\cos(2h)}{h}$

d. $\frac{1 - \sin(2h)}{h}$

**30.** The relative approximate error at the end of an iteration to find the root of an equation is 0·003%. The least number of significant digits we can trust in the solution is

a. 1

b. 2

c. 5

d. 4

**31.** The regula falsi method of finding roots of nonlinear equations falls under the category of _____ methods.

a. open

b. random

c. bracketing

d. graphical

**32.** The order of convergence of the modified Newton's method to find a multiple root with multiplicity 10 of a nonlinear equation is

a. 2

b. 3

c. 10

d. 1·5

**33.** If Newton's second-degree polynomial, which interpolates the data

| $x$ | 15 | 18 | 22 |
|---|---|---|---|
| $y$ | 24 | 37 | 25 |

is $P_2(x) = a_0 + a_1(x-15) + a_2(x-15)(x-18)$, then the value of $a_1$ is most nearly

a. 2·3555

b. 3·3300

c. 1

d. 4·3333

**34.** If the sum of the mean of first $n$ odd natural numbers with mean of $n$ consecutive even numbers beginning from $2n$ is $n+8$, then $n$ equals to

a. 2

b. 3

c. 4

d. 5

**35.** If in a moderately skewed distribution, the values of the mode and mean are $6\alpha$ and $9\alpha$ respectively, then the value of the median is

a. $8\alpha$

b. $7\alpha$

c. $6\alpha$

d. $5\alpha$

**36.** If a student goes to school by bicycle at a speed of 15 km/hr and returns at a speed of 10 km/hr, then his average speed is

a. 12·5 km/hr

b. 12·3 km/hr

c. 12 km/hr

d. 13 km/hr

**37.** Expectation is a generalization of

a. arithmetic mean

b. median

c. mode

d. None of the above

**38.** Slack is defined as

a. the difference between the left and right sides of a constraint

b. the amount by which the left side of $a \geq$ constraint is larger than the right side

c. any variable in a linear programming problem

d. the amount by which the left side of $a \leq$ constraint is smaller than the right side

**39.** The set of feasible solutions in a linear programming problem is a

a. disconnected set

b. non-convex set

c. convex set

d. None of the above

**40.** For an LPP, which of the following is the correct relation between the number of basic feasible solutions (BFS) and the number of vertices?

a. Number of BFS $\leq$ Number of vertices

b. Number of BFS $\geq$ Number of vertices

c. Number of BFS = Number of vertices

d. None of the above

## PART—B

**41.** The limit of the sequence $\{a_n\}$, where

$$a_n = \left(\frac{n+1}{n-1}\right)^n$$

is

a. $e$

b. $e^2$

c. 1

d. 2

**42.** The series

$$\sum_{n=1}^{\infty} \frac{(x+\pi)^n}{\sqrt{n}}$$

converges when

a. $-1-\pi \le x < (1-\pi)$

b. $-1-\pi \le x \le (1-\pi)$

c. $-1-\pi < x < \pi$

d. $-1 \le x \le (1+\pi)$

**43.** The integral $\iint_R e^{x^2+y^2}\, dy\, dx$ where $R$ is the semicircular region bounded by the $x$-axis and the curve $y = \sqrt{1-x^2}$ equals

a. $\frac{\pi}{2}(e+1)$

b. $\frac{\pi}{2}(e-1)$

c. $\frac{\pi}{2}e^2$

d. $\frac{\pi}{2}e$

**44.** If $f'' > 0$ throughout an interval $[a, b]$, then $f'$ has

a. exactly two zeros in $[a, b]$

b. exactly one zero in $[a, b]$

c. at most one zero in $[a, b]$

d. at least one zero in $[a, b]$

45. If $f$ is differentiable on [0, 1] and its derivative is never zero, then

a. $f(0) \leq f(1)$

b. $f(0) \geq f(1)$

c. $f(0) = f(1)$

d. $f(0) \neq f(1)$

46. If $|z - i| \leq 2$ and $z_0 = 5 + 3i$, then the maximum value of $|iz + z_0|$ is

a. 7

b. $2 + \sqrt{31}$

c. $\sqrt{31} - 2$

d. 4

47. If $A(z_1)$, $B(z_2)$ and $C(z_3)$ are three points in the Argand plane such that $z_1 + wz_2 + w^2 z_3 = 0$, $w$ is a cube root of unity, then

a. $A$, $B$, $C$ are collinear

b. $\Delta ABC$ is a right-angled triangle

c. $\Delta ABC$ is an equilateral triangle

d. $\Delta ABC$ is a right-angled isosceles triangle

48. If $a, b, c, d$ and $p$ are distinct real numbers such that

$$(a^2 + b^2 + c^2)p^2 - 2p(ab + bc + cd) + (b^2 + c^2 + d^2) \leq 0$$

then $a, b, c, d$ are in

a. arithmetic progression

b. geometric progression

c. harmonic progression

d. $ab = cd$

**49.** If $|a| < 1$ and $|b| < 1$, then the sum of the series

$$1 + (1+a)b + (1+a+a^2)b^2 + (1+a+a^2+a^3)b^3 + \cdots$$

is

a. $\dfrac{1}{(1-a)(1-b)}$

b. $\dfrac{1}{(1-a)(1-ab)}$

c. $\dfrac{1}{(1-a)(1-b)(1-ab)}$

d. $\dfrac{1}{(1-b)(1-ab)}$

**50.** The sum of the series

$$\frac{3}{4} + \frac{15}{16} + \frac{63}{64} + \cdots$$

up to $n$ terms is

a. $n + \dfrac{4^{-n}}{3} - \dfrac{1}{3}$

b. $n - \dfrac{4^{-n}}{3} - \dfrac{1}{3}$

c. $n + \dfrac{4^{n}}{3} - \dfrac{1}{3}$

d. $n + \dfrac{4^{-n}}{3} + \dfrac{1}{3}$

**51.** If $f: R \to R$ be a differentiable function such that $f(x+2y) = f(x) + f(2y) + 4xy$ for all $x, y \in R$, then

a. $f'(1) = f'(0) + 1$

b. $f'(0) = f'(1) - 2$

c. $f'(1) = f'(0) - 1$

d. $f'(0) = f'(1) + 2$

52. In [0, 1], Lagrange's mean value theorem is not applicable to

a. $f(x) = x|x|$

b. $f(x) = \begin{cases} \frac{\sin x}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$

c. $f(x) = \begin{cases} \frac{1}{2} - x, & x < \frac{1}{2} \\ (\frac{1}{2} - x)^2, & x \geq \frac{1}{2} \end{cases}$

d. $f(x) = |x|$

53. Consider the polynomial $f(x) = 1 + 2x + 3x^2 + 4x^3$. Let $s$ be the sum of all distinct real roots of $f(x)$. Then $s$ lies in the interval

a. $\left(\frac{-1}{4}, 0\right)$

b. $\left(-11, \frac{-3}{4}\right)$

c. $\left(\frac{-3}{4}, \frac{-1}{2}\right)$

d. $\left(0, \frac{1}{4}\right)$

54. If the minimum value of $a\tan^2 x + b\cot^2 x$ equals the maximum value of $a\sin^2 x + b\cos^2 x$, where $a > b > 0$, then

a. $a = b$

b. $a = 2b$

c. $a = 3b$

d. $a = 4b$

55. Let $g(x)$ be differentiable function satisfying $\frac{d}{dx} g(x) = g(x)$ and $g(0) = 1$, then

$$\int g(x)\left(\frac{2 - \sin 2x}{1 - \cos 2x}\right) dx$$

is equal to

a. $g(x)\cot x + c$

b. $-g(x)\cot x + c$

c. $\frac{g(x)}{1 - \cos 2x} + c$

d. None of the above

**56.** Let $A$ be an $n*n$ matrix from the set of numbers $A^3 - 3A^2 + 4A - 6I = 0$, where $I$ is $n*n$ unit matrix. If $A^{-1}$ exists, then $A^{-1} =$

a. $\frac{1}{6}(A^2 - 3A + 4I)$

b. $\frac{1}{6}(A^2 - 3A - 4I)$

c. $\frac{1}{6}(-A^2 - 3A + 4I)$

d. $\frac{1}{6}(-A^2 + 3A - 4I)$

**57.** Let $T$ be an arbitrary linear transformation from $R^n$ to $R^n$ which is not one-one. Then

a. Rank $T > 0$

b. Rank $T = n$

c. Rank $T < n$

d. Rank $T = n - 1$

**58.** If matrix $A$ has an inverse $B$ and $C$, then

a. $B = C$

b. $B \neq C$

c. $B = nC$, for any $n$

d. None of the above

**59.** In an inner product space $V$, which one of the following statements is true?

a. Every orthonormal set in $V$ must be a basis for $V$

b. Every orthonormal set in $V$ must be linearly independent in $V$ but need not necessarily be a basis for $V$

c. Every orthonormal set in $V$ must span $V$ but need not necessarily be linearly independent

d. Every orthonormal set in $V$ must be finite

**60.** Let $V$ be a vector space over a field $F$ and $\dim V = n$. Let $S$ be a subset of $V$ having $n$ vectors. Consider the following statements :

(I) $S$ is a basis for $V$.

(II) $S$ is a spanning set for $V$.

(III) $S$ is linearly independent.

Then

a. (I) $\Leftrightarrow$ (II) is true, but (II) $\Leftrightarrow$ (III) is false

b. (I) $\Leftrightarrow$ (III) is true, but (II) $\Leftrightarrow$ (III) is false

c. (II) $\Leftrightarrow$ (III) is true, but (I) $\Leftrightarrow$ (II) is false

d. (I) $\Rightarrow$ (II) and (I) $\Rightarrow$ (III)

**61.** The set $\{x^2 + 4x - 3, 2x^2 + x + 5, 7x - 11\}$ of $\mathbb{P}_2$

a. spans $\mathbb{P}_2$

b. spans and is a basis of $\mathbb{P}_2$

c. does not span $\mathbb{P}_2$

d. None of the above

**62.** Let $G$ be a group of order 45. Let $H$ be a 3-Sylow subgroup of $G$ and $K$ be a 5-Sylow subgroup of $G$. Then

a. $H$ is normal in $G$ but $K$ is not normal in $G$

b. both $H$ and $K$ are normal in $G$

c. $H$ is not normal in $G$ but $K$ is normal in $G$

d. both $H$ and $K$ are not normal in $G$

63. Let $C^*$ be a set of non-zero complex numbers. Under the operations of multiplication, $C^*$ forms a group. If $z = a + ib$ is a complex number, then the inverse of $z$ in $C^*$ is

a. $\frac{a - ib}{a^2 + b^2}$

b. $\frac{a + ib}{a^2 + b^2}$

c. $\frac{a - ib}{a^2 - b^2}$

d. $\frac{a + ib}{a^2 - b^2}$

64. In a group $G$, which of the following is ***not*** true?

a. $g^m g^n = g^{m+n} \ \forall \ m, n \in \mathbb{Z}$

b. $(g^m)^n = g^{mn} \ \forall \ m, n \in \mathbb{Z}$

c. $(gh)^n = (h^{-1} g^{-1})^{-n} \ \forall \ n \in \mathbb{Z}$

d. $(gh)^n = (g^{-1} h^{-1})^{-n} \ \forall \ n \in \mathbb{Z}$

65. The permutation

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 6 & 3 & 5 & 1 & 4 & 2 & 7 \end{pmatrix}$$

is a cycle of length

a. 2

b. 4

c. 3

d. 6

66. Which one of the following groups has a proper subgroup that is not cyclic?

a. $Z_{15} \times Z_{17}$

b. $(Q, +)$

c. $S_3$

d. $(Z, +)$

**67.** If $R$ is a commutative ring with unit element, $M$ is an ideal of $R$ and $R/M$ is a finite integral domain, then

a. $M$ is a maximal ideal of $R$

b. $M$ is not a maximal ideal of $R$

c. $M$ is a minimal ideal of $R$

d. None of the above

**68.** The number of prime ideals of $Z_{10}$ is

a. 1

b. 0

c. 2

d. 5

**69.** If $U(m)$ is the set of units of $Z_m$, then $U(2^n)$, where $n \geq 3$, is

a. cyclic

b. Abelian

c. Abelian but not cyclic

d. None of the above

**70.** The order of Aut($G$), where $G$ is a group with 65 elements, is

a. 1

b. 2

c. 64

d. 48

71. The partial differential equation

$$\frac{\partial^2 u}{\partial x^2} + x\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y} = 0$$

is classified as

a. elliptic when $x < 0$ and hyperbolic when $x < 0$

b. elliptic when $x > 0$ and hyperbolic when $x < 0$

c. elliptic when $x > 0$ and hyperbolic when $x > 0$

d. elliptic and hyperbolic when $x < 0$

72. Which one of the following satisfies the partial differential equation $\frac{\partial^3 z}{\partial x^3} - 3\frac{\partial^3 z}{\partial x^2 \partial y} + 4\frac{\partial^3 z}{\partial y^3} = e^{x+2y}$?

a. $z(x, y) = f_1(y-x) + f_2(y+2x) + xf_3(y+2x) + \frac{1}{27}e^{x+2y}$

b. $z(x, y) = f_1(y-x) + f_2(y+2x) + \frac{1}{27}e^{x+2y}$

c. $z(x, y) = f_1(y-x) + xf_2(y+2x) + \frac{1}{27}e^{x+2y}$

d. $z(x, y) = f_1(y-x) + f_2(y+2x) + x^2 f_3(y+2x)$

73. The solution of an ordinary differential equation

$$\frac{dy}{dx} = \frac{x-y+3}{2x-2y+5}$$

is

a. $(x-5y) + \log(x-y+2) = c$

b. $(x+2y) + \log(x-y+2) = c$

c. $(x-2y) + \log(x+y+2) = c$

d. $(x-2y) + \log(x-y+2) = c$

**74.** Which one of the following differential equations represents the family of curves $y(x) = Ae^{2x} + Be^{-2x}$, where $A$, $B$ are constants?

a. $\frac{d^2y}{dx^2} - 2y = 0$

b. $\frac{d^2y}{dx^2} + 4y = 0$

c. $\frac{d^2y}{dx^2} - 4y = 0$

d. $\frac{d^2y}{dx^2} + 2y = 0$

**75.** D'Alembert's solution of the one-dimensional wave equation is

a. $u(x, t) = f(x + ct) + g(x - ct)$

b. $u(x, t) = f(x + ct) + g(x + 2ct)$

c. $u(x, t) = f(x + 2ct) + g(x - ct)$

d. $u(x, t) = f(x + ct) + g(x^2 - ct)$

**76.** The Laplace equation in polar coordinates is

a. $\frac{\partial^2 u}{\partial r^2} + r\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial \theta^2} = 0$

b. $r^2\frac{\partial^2 u}{\partial r^2} + r\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial \theta^2} = 0$

c. $r^2\frac{\partial^2 u}{\partial r^2} + \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial \theta^2} = 0$

d. $r^2\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial \theta^2} = 0$

77. The general solution of the differential equation

$$\frac{d^4 y}{dx^4} - y = \sin(x)$$

is

a. $y(x) = c_1 e^{x} + c_2 e^{-2x} + c_3 \cos(x) + c_4 \sin(x) + \frac{x}{4}\cos(x)$

b. $y(x) = c_1 e^{x} + c_2 e^{-x} + c_3 \cos(x) + c_4 \sin(x) + \frac{x}{4}\cos(x)$

c. $y(x) = c_1 e^{2x} + c_2 e^{-x} + c_3 \cos(x) + c_4 \sin(x) + \frac{x}{4}\cos(x)$

d. $y(x) = c_1 e^{x} + c_2 e^{-x} + c_3 \cos(x) + c_4 \sin(2x) + \frac{x}{4}\cos(2x)$

78. The solution of a partial differential equation

$$\left(\frac{x}{z}\frac{\partial z}{\partial x}\right)^2 + \left(\frac{y}{z}\frac{\partial z}{\partial y}\right)^2 = 1$$

is

a. $\log(z) = A\log(2x) + \sqrt{1-A^2}\,\log(y) + B$

b. $\log(z) = A\log(x) + \sqrt{1-A^2}\,\log(2y) + B$

c. $\log(z) = A\log(x) + \sqrt{1-A^2}\,\log(y) + B$

d. $\log(z) = A\log(x) + \sqrt{1-A^2}\,\log(3y) + B$

79. The solution of a first-order partial differential equation $\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}$, given $u(0, y) = 8e^{-3y}$ is

a. $u(x, y) = 8e^{-12x-3y}$

b. $u(x, y) = 3e^{-12x-3y}$

c. $u(x, y) = 3e^{-3x-12y}$

d. $u(x, y) = 8e^{-x-3y}$

80. In two-dimensional heat flow, the temperature along the normal to the $XY$-plane is

a. infinity

b. one

c. indeterminate

d. zero

81. The complementary function of the differential equation

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - 4y = x^2$$

is

a. $Ax^4 + Bx^{-4}$

b. $Ax^2 + Bx^{-2}$

c. $Ax + Bx^{-2}$

d. $Ax^2 + Bx^{-1}$

82. The solution of the initial value problem $\frac{d^2y}{dx^2} = 2(y + y^3)$ under the condition $y = 0$, $\frac{dy}{dx} = 1$, when $x = 0$ is

a. $y(x) = \tan(x)$

b. $y(x) = \sin(x)$

c. $y(x) = \cos(x)$

d. $y(x) = \tan(2x + 1)$

83. The subsidiary equation associated with Lagrange's linear partial differential equation

$$P(x, y, z)\frac{\partial z}{\partial x} + Q(x, y, z)\frac{\partial z}{\partial y} = R(x, y, z)$$

is

a. $\frac{dx}{P+1} = \frac{dy}{Q} = \frac{dz}{R}$

b. $\frac{dx}{P} = \frac{dy}{Q+1} = \frac{dz}{R}$

c. $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R+1}$

d. $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$

84. Consider a nonlinear equation $x^2 - 2x - 3 = 0$.

a. The sequence generated by $x_{i+1} = \frac{x_i^2 - 3}{2}$ converges to a root of the above nonlinear equation

b. The sequence generated by $x_{i+1} = \frac{3}{x_i - 2}$ does not converge to a root of the above nonlinear equation

c. The sequence generated by $x_{i+1} = \frac{x_i^2 + 3}{2(x_i - 1)}$ converges quadratically to a root of the above nonlinear equation

d. The sequence generated by $x_{i+1} = \sqrt{2x_i + 3}$ does not converge to a root of the above nonlinear equation

85. A wooden block is measured to be 60 mm by a ruler and the measurements are considered to be good to $\frac{1}{4}$th of a millimeter. Then in the measurement of 60 mm, we have _____ significant digits.

a. 1

b. 5

c. 2

d. 4

**86.** If in the calculation of the volume of a cube of nominal size 5″, the uncertainty in the measurement of each side is 10%, then the uncertainty in the measurement of the volume would be

a. 30%

b. 15%

c. 10·5%

d. 15·35%

**87.** The secant method formula for finding the square root of a real number $R$ from the equation $x^2 - R = 0$ is

a. $\dfrac{x_i x_{i-1} - R}{x_i + x_{i-1}}$

b. $\dfrac{x_i x_{i-1} + R}{x_i + x_{i-1}}$

c. $\dfrac{x_i x_{i-1}}{x_i + x_{i-1}}$

d. $\dfrac{x_i^2 - R}{x_i + x_{i-1}}$

**88.** Given the two points $(x_0,\ y(x_0))$ and $(x_1,\ y(x_1))$. The linear Lagrange polynomial $P_1(x)$ that passes through these two points is given by

a. $P_1(x) = \dfrac{x - x_0}{x - x_1}\, y(x_0) + \dfrac{x - x_1}{x_1 - x_0}\, y(x_1)$

b. $P_1(x) = \dfrac{x - x_0}{x - x_1}\, y(x_0) + \dfrac{x - x_1}{x - x_0}\, y(x_1)$

c. $P_1(x) = \dfrac{x - x_0}{x_0 - x_1}\, y(x_0) + \dfrac{x - x_1}{x_1 - x_0}\, y(x_1)$

d. $P_1(x) = \dfrac{x - x_1}{x_0 - x_1}\, y(x_0) + \dfrac{x - x_0}{x_1 - x_0}\, y(x_1)$

89. In an attempt to understand the mechanism of the depolarization process in a fuel cell, an electrokinetic model for mixed oxygen-methanol current on platinum was developed in the laboratory at FAMU. A very simplified model of the reaction developed suggests a functional relation in an integral form. To find the time required for 50% of the oxygen to be consumed, the time, $T(s)$ is given by

$$T = -\int_{1\cdot22\times10^{-6}}^{0\cdot61\times10^{-6}} \left( \frac{6\cdot73x + 4\cdot3025\times10^{-7}}{2\cdot316\times10^{-11}x} \right) dx$$

The time required for 50% of the oxygen to be consumed using Simpson's $\frac{1}{3}$rd rule will be nearest to

a. 185160 s

b. 180160 s

c. 190160 s

d. 195290 s

90. The division by zero during forward elimination steps in Gaussian elimination of the system of linear equations implies that the coefficient matrix

a. is singular

b. may be singular or non-singular

c. is invertible

d. is non-singular

91. The highest order of polynomial integrand, for which the trapezoidal rule of integration is exact, is

a. first

b. second

c. third

d. fourth

92. The value of $c$, for which $p(x = k) = ck^2$ can serve as the probability function of a random variable $X = \{1, 2, 3, 4, 5\}$, is

a. $\frac{1}{5}$

b. $\frac{1}{10}$

c. $\frac{1}{15}$

d. $\frac{1}{55}$

93. In a binomial distribution with $n$ number of trials, the probability of success in a trial is $\frac{1}{4}$. If the probability of at least one success is greater than or equal to $\frac{9}{10}$, then $n$ is greater than

a. $\dfrac{9}{\log_{10} 4 - \log_{10} 3}$

b. $\dfrac{1}{\log_{10} 4 + \log_{10} 3}$

c. $\dfrac{1}{\log_{10} 4 - \log_{10} 3}$

d. $\dfrac{4}{\log_{10} 4 - \log_{10} 3}$

94. A student applies in three universities $A$, $B$ and $C$ for admission. His chance of selection is 20%, 16% and 14% in universities $A$, $B$ and $C$ respectively. The chance of selection in both $A$ and $B$ is 8%, $A$ and $C$ is 5%, $B$ and $C$ is 4% and in all $A$, $B$ and $C$ is 2%. The chance of the selection of the student in at least one university is

a. 0·3

b. 0·35

c. 0·4

d. 0·45

95. The diameter, say $X$, of an electric cable is assumed to be continuous random variable with probability density function $f(x) = x(1-x);\ 0 \le x \le 1$. Determine the value of $k$ such that $P(x > k) = p(x < k)$, is

a. $\dfrac{1}{4}$

b. $\dfrac{1+\sqrt{3}}{2}$

c. $\dfrac{1-\sqrt{3}}{2}$

d. $\dfrac{1}{2}$

96. If all the constraints in an LPP are expressed as equalities, the problem is said to be written in

a. standard form

b. bounded form

c. feasible form

d. alternative form

**97.** For an LPP, if in a simplex table, the relative cost $z_j - c_j$ is zero for a non-basic variable, then there exists an alternate optimal solution, provided

a. it is any simplex table

b. it is a starting simplex table

c. it is an optimal simplex table

d. None of the above

**98.** For an LPP, the amount by which an objective function coefficient can change before a different set of values for the decision variables becomes optimal is the

a. range of feasibility

b. range of optimality

c. optimal solution

d. dual solution

**99.** The dual of the LPP $\min c^T x$ subject to $Ax \geq b$ and $x \geq 0$ is

a. $\max b^T w$ subject to $A^T w \geq c$ and $w \geq 0$

b. $\max b^T w$ subject to $A^T w \leq c$ and $w \geq 0$

c. $\max b^T w$ subject to $A^T w \leq c$ and $w$ is unrestricted

d. $\max b^T w$ subject to $A^T w \geq c$ and $w$ is unrestricted

**100.** Which one of the following statements is correct?

a. If an LPP is infeasible, then its dual is also infeasible.

b. If an LPP is infeasible, then its dual always has unbounded solution.

c. If an LPP has unbounded solution, then its dual also has unbounded solution.

d. If an LPP has unbounded solution, then its dual is infeasible.

SPACE FOR ROUGH WORK

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