Test Centre : ____________________

Roll No. : ____________________

Name of the Candidate : ____________________

S A U

Entrance Test for M.Sc. (Applied Mathematics) 2018

[PROGRAMME CODE : 30005]

Question Paper Series Code : A

QUESTION PAPER

Time : 3 hours *Maximum Marks* : 100

INSTRUCTIONS FOR CANDIDATES

Read carefully the following instructions before attempting the Question Paper :

(i) Write your Name, Roll Number and Name of the Test Centre in the space provided for the purpose on the top of this Question Paper and on the OMR Sheet.

(ii) This Question Paper has Two Parts : Part—A and Part—B.

(iii) Part—A (objective-type) has 40 questions of **1** mark each. All questions are compulsory. Part—B (objective-type) has 60 questions of **1** mark each. All questions are compulsory.

(iv) **A wrong answer will lead to the deduction of one-fourth ($\frac{1}{4}$) of the marks assigned to that question.**

(v) Symbols have their usual meanings.

(vi) **Please darken the appropriate circle of 'Question Paper Series Code' and 'Programme Code' on the OMR Sheet in the space provided.**

(vii) All questions should be answered on the OMR Sheet.

(viii) Answers written inside the Question Paper will **NOT** be evaluated.

(ix) **Non-programmable calculators and log tables may be used. Mobile phones are NOT allowed.**

(x) Pages at the end of the Question Paper have been provided for Rough Work.

(xi) **Return the Question Paper and the OMR Sheet** to the Invigilator at the end of the Entrance Test.

(xii) **DO NOT FOLD THE OMR SHEET.**

INSTRUCTIONS FOR MARKING ANSWERS ON THE 'OMR SHEET'

Use BLUE/BLACK Ballpoint Pen Only

1. **Please ensure that you have darkened the appropriate circle of 'Question Paper Series Code' and 'Programme Code' on the OMR Sheet in the space provided.**

   Question Paper Series Code

   Write Question Paper Series Code **A** or **B**
   in the box and darken the appropriate circle.

   ☐ **A or B**

   ●

   Ⓑ

2. Use only Blue/Black Ballpoint Pen to darken the circle. Do not use Pencil to darken the circle for Final Answer.
3. Please darken the whole circle. ●
4. Darken ONLY ONE CIRCLE for each question as shown below in the example :

   ***Example* :**

Wrong	Wrong	Wrong	Wrong	Correct
● ⓑ ⓒ ●	⊘ ⓑ ⓒ ⓓ	⊘ ⓑ ⓒ ⊘	⊙ ⓑ ⓒ ●	ⓐ ⓑ ⓒ ●

5. Once marked, no change in the answer is possible.
6. Please do not make any stray marks on the OMR Sheet.
7. Please do not do any rough work on the OMR Sheet.
8. Mark your answer only in the appropriate circle against the number corresponding to the question.
9. **A wrong answer will lead to the deduction of one-fourth of the marks assigned to that question.**
10. Write your seven-digit Roll Number in small boxes provided for the purpose; and also darken the appropriate circle corresponding to respective digits of your Roll Number as shown in the example below.

    ***Example* :**

ROLL NUMBER

1	3	5	7	2	0	2
●	①	①	①	①	①	①
②	②	②	②	●	②	●
③	●	③	③	③	③	③
④	④	④	④	④	④	④
⑤	⑤	●	⑤	⑤	⑤	⑤
⑥	⑥	⑥	⑥	⑥	⑥	⑥
⑦	⑦	⑦	●	⑦	⑦	⑦
⑧	⑧	⑧	⑧	⑧	⑧	⑧
⑨	⑨	⑨	⑨	⑨	⑨	⑨
⓪	⓪	⓪	⓪	⓪	●	⓪

PART—A

1. $\lim\limits_{x \to 0} \dfrac{\sin 3x}{\tan 5x}$ is equal to

   a. $\dfrac{3}{5}$

   b. $\dfrac{5}{3}$

   c. 0

   d. 1

2. The function $f(x) = 3x^4 - 4x^3$ is concave upward in

   a. $\left(0, \dfrac{2}{3}\right)$

   b. $\left(\dfrac{2}{3}, \infty\right)$

   c. $(-\infty, 0) \cup \left(\dfrac{2}{3}, \infty\right)$

   d. $(0, \infty)$

3. The number in the interval $[0 \cdot 5, 1 \cdot 5]$ such that the sum of the number and its reciprocal is the largest is

   a. 0·5

   b. 0·7

   c. 0·9

   d. 1·4

4. The infinite series

$$\sum_{n=1}^{\infty} \frac{n!}{(2n)!} x^n$$

   converges for

   a. $x < 4$

   b. $x \leq 4$

   c. $x > 4$

   d. $x \geq 4$

5. Which one of the following statements is true?

a. Lagrange's mean value theorem can be deduced from Rolle's theorem.

b. Rolle's theorem can be deduced from Lagrange's mean value theorem.

c. Cauchy mean value theorem can be deduced from Lagrange's mean value theorem.

d. Cauchy mean value theorem can be deduced from Rolle's theorem.

6. The function $f(x) = 4 - 3x - x^2$ is decreasing in the interval

a. $\left(-\frac{3}{2}, \infty\right)$

b. $\left(-\infty, -\frac{3}{2}\right)$

c. $\left(-\frac{3}{2}, 0\right)$

d. $\left(\frac{3}{2}, \infty\right)$

7. If the function

$$f(x) = \begin{cases} x^2, & x \le 0 \\ x^3, & x > 0 \end{cases}$$

then the value of $f''(0)$ is

a. 0

b. 2

c. 3

d. Does not exist

8. If the function

$$f(x) = \begin{cases} 3x^2, & x \le 1 \\ ax + b, & x > 1 \end{cases}$$

then the values of a and b so that f is differentiable at $x = 1$, are

a. –2, 5

b. 2, –5

c. 2, 5

d. –2, –5

9. $\lim_{n\to\infty}\left[\frac{1}{n^2}+\frac{1}{(n+1)^2}+\cdots+\frac{1}{(2n)^2}\right]$ is equal to

a. 0

b. 1

c. 4

d. Does not exist

10. $\lim_{n\to\infty}\frac{(3n)!}{(n!)^3}$ is equal to

a. $\frac{1}{27}$

b. $\frac{1}{9}$

c. 9

d. 27

11. If A is a 5×5 matrix with real entries, then A has

a. an eigenvalue which is purely imaginary

b. at least one real eigenvalue

c. at least two eigenvalues which are not real

d. at least two distinct real eigenvalues

12. If A is an $n\times n$ matrix with real entries such that $A^k = 0$ (0-matrix), for some $k\in N$, then

a. A has to be the 0 matrix

b. trace (A) could be non-zero

c. A is diagonalizable

d. 0 is the only eigenvalue of A

13. If T is a linear transformation from an n-dimensional vector space U to an m-dimensional vector space V, then the sum of the rank of T and the nullity of T is equal to

a. n

b. m

c. $n-m$

d. $n+m$

14. If T_1 and T_2 are linear operators on R^2 defined as

$$T_1(a, b) = (b, a) \text{ and } T_2(a, b) = (a, 0)$$

then T_1T_2 defined by $T_1T_2(a, b) = T_1(T_2(a, b))$ maps (1, 2) into

a. (1, 0)

b. (0, 1)

c. (2, 0)

d. (0, 2)

15. If $G = Z_{100}$, then the number of elements of order 50 in G is

a. 20

b. 25

c. 30

d. 50

16. How many proper subgroups does the group $Z \oplus Z$ have?

a. 1

b. 2

c. 3

d. Infinitely many

17. What is the last digit of 97^{2018}?

a. 1

b. 3

c. 7

d. 9

18. The order of Aut(G), where G is a group with 17 elements is

a. 1

b. 16

c. 17

d. 2

19. In $Z[x]$, the ideal of $< x >$ is

a. maximal but not prime

b. prime but not maximal

c. both prime and maximal

d. neither prime nor maximal

20. There exists a finite field of order

a. 6

b. 12

c. 16

d. 24

21. The solution of the differential equation $16y\frac{dy}{dx} + 9x = 0$, represents

a. a parabola

b. an ellipse

c. a hyperbola

d. a rectangle

22. Which one of the following satisfies the differential equation $x\frac{dy}{dx}+\frac{y^2}{x}=y$?

a. $y+\log(cx)=0,\ c=$ constant

b. $x+\log(cx)=0,\ c=$ constant

c. $\frac{y}{x}+\log(cx)=0,\ c=$ constant

d. $\frac{x}{y}+\log(cx)=0,\ c=$ constant

23. Which one of the following is the integrating factor to the differential equation $\sin(2x)\frac{dy}{dx}=y+\tan(x)$?

a. $\frac{1}{\sqrt{\tan(x)}}$

b. $\frac{1}{\sqrt{\sin(x)}}$

c. $\frac{1}{\sqrt{\cos(x)}}$

d. $\frac{1}{\sqrt{\tan(2x)}}$

24. A real-valued function $f: D \to R$ defined on the connected open set D in R^2 is said to satisfy the Lipschitz condition in y on D with Lipschitz constant M if and only if

a. $|f(y_2)-f(y_1)| \le M|y_2|,\ \forall (x, y_1), (x, y_2) \in D$

b. $|f(x, y_2)-f(x, y_1)| \le M|y_1|,\ \forall (x, y_1), (x, y_2) \in D$

c. $|f(x, y_2)-f(x, y_1)| \le M|y_2-y_1|,\ \forall (x, y_1), (x, y_2) \in D$

d. $|f(x, y_2)-f(x, y_1)| \le M,\ \forall (x, y_1), (x, y_2) \in D$

25. If the population of a city gets doubled in two years and after three years the population is 15000. What is the initial population of the city? [Given $\log(2) = 0 \cdot 693, e^{1 \cdot 041} = 2 \cdot 832$]

a. 0

b. 4297

c. 5498

d. 5297

26. If

$$M(x, y)\,dx + N(x, y)\,dy = 0 \text{ and } \frac{1}{M}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) = \Phi(y)$$

then the integrating factor is

a. $M(x, y)$

b. $e^{\int \Phi(y)\,dy}$

c. $N(x, y)$

d. $M(x, y) + N(x, y)$

27. If the differential equation

$$9x(1-x)\frac{d^2y}{dx^2} - 12\frac{dy}{dx} + 4y = 0$$

then the point $x = 0$ is

a. a regular singular point

b. an ordinary point

c. an irregular point

d. a focal point

28. Truncation error is caused by approximating

a. irrational numbers

b. rational numbers

c. exact mathematical procedures

d. fractions

29. The order of convergence of the secant method to find the root of a non-linear equation, is

a. 1·618

b. 3

c. 1

d. 1·5

30. The matrix

$$A = \begin{pmatrix} 2 & 1 \\ 2 & 1\cdot 01 \end{pmatrix}$$

is

a. well-conditioned

b. singular

c. non-singular

d. ill-conditioned

31. In the _____ method, a system is reduced to an equivalent diagonal form using elementary transformations.

a. Jacobi

b. Gauss-Jordan

c. Gauss elimination

d. Gauss-Seidel

32. The highest order of polynomial integrand for which Simpson's $\frac{1}{3}$rd rule of integration is exact, is

a. first

b. second

c. third

d. fourth

33. Given $n+1$ data pairs, a unique polynomial of degree ____ passes through the $n+1$ data points.

a. n or less

b. n

c. $n+1$

d. $n+1$ or less

34. We are given a box containing 5000 transistors of which 1000 are manufactured by company SUPER and rest by company EXCELLENT. Ten percent of the transistors made by company SUPER and five percent of the transistors made by company EXCELLENT are defective. If a randomly chosen transistor is found to be defective, what is the probability that it is of company SUPER?

a. $\frac{2}{5}$

b. $\frac{1}{5}$

c. $\frac{1}{3}$

d. $\frac{3}{5}$

35. If a, b, c are three distinct positive real numbers selected from the open interval $I = (0, 10)$, then the probability of finding the least value of expression

$$\frac{(1+a+a^2)(1+b+b^2)(1+c+c^2)}{abc}$$

in I is

a. 1

b. 0

c. Cannot be determined because information given is insufficient

d. None of the above

36. The random variable X is finite, discrete and it is uniformly distributed. The coefficient of variation $C_X = \frac{\text{Variance}}{(E(X))^2}$ of X is

a. $0 \le C_X < \frac{1}{4}$

b. $0 \le C_X < \frac{1}{2}$

c. $0 \le C_X < \frac{1}{\sqrt{12}}$

d. $0 \le C_X < \frac{1}{\sqrt{3}}$

37. Consider 5 independent Bernoulli's trials each with probability of success p. If the probability of at least one failure is greater than or equal to $\frac{31}{32}$, then p lies in the interval

a. $\left(\frac{1}{2}, \frac{3}{4}\right]$

b. $\left(\frac{3}{4}, \frac{11}{12}\right]$

c. $\left[0, \frac{1}{2}\right]$

d. $\left(\frac{11}{12}, 1\right]$

38. The intersection of a finite number of closed half spaces in R^n is called a

a. convex set

b. polyhedral convex set

c. convex cone

d. closed set

39. A simplex is defined as an n-dimensional convex polyhedron having

a. exactly n vertices

b. exactly $n+1$ vertices

c. vertices $\leq n+1$

d. None of the above

40. The constraints of a linear programming problem with non-negative restrictions are

a. closed half-spaces only

b. hyperplanes only

c. either closed half-spaces or hyperplanes

d. None of the above

PART—B

41. If the sequence defined by

$$x_1 = 1$$
$$x_{n+1} = \frac{4+3x_n}{3+2x_n}, \quad n \geq 1$$

the sequence $\{x_n\}$ converges to

a. $\frac{7}{5}$

b. $\frac{5}{7}$

c. $\sqrt{2}$

d. Does not converge

42. Which one of the following statements is true about the series

$$\sum_{n=1}^{\infty} \frac{m(m-1)\cdots(m-n+1)}{(n-1)!} x^n ?$$

a. The series is absolutely convergent for all x

b. The series is absolutely convergent for $x \in [-1, 1]$

c. The series is absolutely convergent for $x \in (-1, 1)$

d. The series is only convergent for $x \in (-1, 1)$

43. If f is a function defined on a closed interval $[a, b]$, then

a. f is continuous if and only if f is uniformly continuous

b. f may be continuous but need not be uniformly continuous

c. f may be uniformly continuous but need not be continuous

d. There is no relation between continuity and uniform continuity

44. If f is a twice differentiable function satisfying $f(1) = 1$, $f(2) = 4$ and $f(3) = 9$, then

a. $f''(x) = 2, \forall x \in (-\infty, \infty)$

b. $f''(x) = 5 = f'(x)$ for some $x \in [1, 3]$

c. there exists at one $x \in (1, 3)$ such that $f''(x) = 2$

d. None of the above

45. If $f(x)$ is a function defined as

$$f(x) = \begin{cases} \sin(x^2 - 3x), & x \le 0 \\ 6x + 5x^2, & x > 0 \end{cases}$$

then at $x = 0$, $f(x)$

a. has a local maximum

b. has a local minimum

c. is discontinuous

d. None of the above

46. If $f(x)$ is a function defined by $f(x) = \int_1^x t(t^2 - 3t + 2)\,dt$, $x \in [1, 3]$, then the range of $f(x)$ is

a. $[0, 2]$

b. $\left[-\frac{1}{4}, 4\right]$

c. $\left[-\frac{1}{4}, 2\right]$

d. None of the above

47. If $xy = a^2$ and $S = b^2x + c^2y$ where a, b and c are constants, then the minimum value of S is

a. $2abc$

b. $\sqrt{a}bc$

c. abc

d. None of the above

48. $\int(1+x-x^{-1})e^{x+x^{-1}}dx$ is equal to

a. $(x+1)e^{x+x^{-1}}+c$

b. $(x-1)e^{x+x^{-1}}+c$

c. $-xe^{x+x^{-1}}+c$

d. $xe^{x+x^{-1}}+c$

49. $\frac{d}{dx}\int_{x^2}^{x^3}\frac{1}{\log t}\,dt$ is equal to

a. x^2-x

b. $(x^2-x)\log x$

c. $\frac{x^2-x}{\log x}$

d. $\frac{x-1}{\log x}$

50. $\lim\limits_{x\to\infty}\frac{\int_0^{2x}te^{t^2}dt}{e^{4x^2}}$ is equal to

a. 0

b. 2

c. $\frac{1}{2}$

d. ∞

51. Let $f(x)=0$, if x is an irrational number and $f(x)=1$. If x is a rational number, then

a. $f(x)$ is Riemann integrable over rationals

b. $f(x)$ is Riemann integrable on $[0, \infty)$

c. $f(x)$ is not Riemann integrable

d. $f(x)$ is none of the above

52. The integral $I = \int_0^1 x^{m-1}(1-x)^{n-1}dx$ is convergent if

a. $m > 0, n < 0$

b. $m > 0, n > 0$

c. $m < 0, n > 0$

d. $m < 0, n < 0$

53. In the Fourier series representation of the function $f(x) = 1$, the coefficient of sine term is

a. $\frac{2}{n\pi}(1+(-1)^n)$

b. $\frac{2}{n\pi}(1-(-1)^n)$

c. $\frac{2}{n\pi}(-1)^n$

d. $\frac{1}{n\pi}(1-(-1)^n)$

54. The circulation of $\vec{F}$ round the curve C, where $\vec{F} = y\hat{i} + z\hat{j} + x\hat{k}$ and C is the circle $x^2 + y^2 = 1$, $z = 0$, is

a. $-\pi$

b. π

c. $-\pi/2$

d. $\pi/2$

55. For the function $f = \frac{y}{x^2+y^2}$, which one of the following is the value of the directional derivative making an angle of 30° with the positive x-axis at point (0, 1)?

a. $-1/2$

b. 1

c. $1/2$

d. 2

56. If S_{10} denotes the group of permutations on ten symbols $\{1, 2, \cdots, 10\}$, then the number of elements of S_{10} commuting with the element $\sigma = (13579)$, is

a. 5!

b. 5·5!

c. 5!5!

d. $\frac{10!}{5!}$

57. The value of α for which $G = \{\alpha, 1, 3, 9, 19, 27\}$ is a cyclic group under multiplication modulo 56, is

a. 5

b. 15

c. 25

d. 35

58. In the group $(Z, +)$, the subgroup generated by 2 and 7 is

a. Z

b. $5Z$

c. $9Z$

d. $14Z$

59. The cardinality of the centre of Z_{12} is

a. 1

b. 2

c. 3

d. 12

60. If H and K are subgroups of G with indices 3 and 5 in G, then the index of $H \cap K$ in G is

a. a multiple of 15

b. 3

c. 5

d. not more than 8

61. The number of elements of order 5 in the symmetric group S_5, is

a. 5

b. 20

c. 24

d. 12

62. Let G be a group of order 30. Let A and B be normal subgroups of orders 2 and 5 respectively. Then the order of the group $G \backslash AB$ is

a. 10

b. 3

c. 2

d. 5

63. Which one of the following statements is true?

a. $\exists$ a field of order 36 but not order 49

b. $\exists$ a field of order 36

c. $\nexists$ a field of order 36 and 49

d. $\nexists$ a field of order 36

64. If I_1 and I_2 be two ideals of a commutative ring R with identity, then which one of the following is true?

a. $I_1 + I_2$ and $I_1 \cap I_2$ are ideals of R

b. $I_1 + I_2$ is an ideal of R, but $I_1 \cap I_2$ is not an ideal of R

c. $I_1 + I_2$ is not an ideal of R, but $I_1 \cap I_2$ is an ideal of R

d. Neither $I_1 + I_2$ nor $I_1 \cap I_2$ is an ideal of R

65. If p is prime and Z_{p^4} denotes the ring of integers modulo p^4, then the number of maximal ideals in Z_{p^4} is

a. 4

b. 2

c. 3

d. 1

66. Consider the following statements :

1. A ring without unity can have maximal ideal which is not prime.

2. The number of maximal ideals in a commutative ring with unity are always less than or equal to the number of prime ideals.

Then

a. 1 is correct and 2 is incorrect

b. 2 is correct but 1 is incorrect

c. both 1 and 2 are correct

d. both 1 and 2 are incorrect

67. Let R be a ring. If $R[x]$ is a principal ideal domain, then R is necessarily

a. a unique factorization domain

b. a principal ideal domain

c. an Euclidean domain

d. a field

68. The union of two subspaces W_1 and W_2 of a vector space $V(F)$ is a subspace, if and only if

a. $W_1 \subset W_2$ and $W_2 \subset W_1$

b. $W_1 \subset W_2$ or $W_2 \subset W_1$

c. $W_1 \not\subset W_2$ and $W_2 \not\subset W_1$

d. None of the above

69. If C^2 [a set of pair of complex numbers] is not a vector space over K, then

a. $K = R$ (a set of real numbers)

b. $K = Q$ (a set of rational numbers)

c. $K = C$ (a set of complex numbers)

d. $K = Z$ (a set of integers)

70. The dimension of the subspace W of vector space R^4, spanned by vectors $(1, -4, -2, 1)$, $(1, -3, -1, 2)$ and $(3, -8, -2, 7)$, is

a. 1

b. 2

c. 3

d. 4

71. Which one of the following satisfies the partial differential equation

$$\left(\frac{1}{z}-\frac{1}{y}\right)\frac{\partial z}{\partial x}+\left(\frac{1}{x}-\frac{1}{z}\right)\frac{\partial z}{\partial y}=\frac{1}{y}-\frac{1}{x}?$$

a. $f(xyz, x+y+z)=0$

b. $f(xy+z, x+y+z)=0$

c. $f(x+yz, x+y+z)=0$

d. $f(xyz, x-y-z)=0$

72. The model equation for the transverse vibrations of a string is

a. $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$

b. $\frac{\partial y}{\partial t} = c^2 \frac{\partial^2 y}{\partial x^2}$

c. $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial y}{\partial x}$

d. $\frac{\partial^2 y}{\partial t^2} + \frac{\partial^2 y}{\partial x^2} = 0$

73. The solution of the partial differential equation

$$\frac{\partial^4 u}{\partial x^4} + \frac{\partial^2 u}{\partial y^4} = 2\frac{\partial^4 u}{\partial x^2 \partial y^2}$$

is

a. $u(x, y) = f_1(x+2y) + xf_2(x-y) + f_3(x+y) + xf_4(x+y)$

b. $u(x, y) = f_1(x-y) + xf_2(x-2y) + f_3(x+y) + xf_4(x+y)$

c. $u(x, y) = f_1(x-y) + xf_2(x-y) + f_3(x+y) + xf_4(x+y)$

d. $u(x, y) = f_1(x-y) + xf_2(x-y) + f_3(x+2y) + xf_4(x+2y)$

74. The solution of the non-homogeneous partial differential equation

$$\frac{\partial^3 u}{\partial x^3} - \frac{\partial^3 u}{\partial y^3} = x^3 y^3$$

is given by

a. $u(x, y) = f_1(y+x) + f_2(y+\omega x) + f_3(y+\omega^2 x) + \frac{x^3 y^6}{120} + \frac{x^9}{10080}$

b. $u(x, y) = f_1(y+x) + f_2(y+\omega x) + f_3(y+\omega^2 x) + \frac{x^6 y^3}{120} + \frac{x^8}{10080}$

c. $u(x, y) = f_1(y+x) + f_2(y+\omega x) + f_3(y+\omega^2 x) + \frac{x^6 y^3}{120} + \frac{x^9}{10080}$

d. $u(x, y) = f_1(y-x) + f_2(y-\omega x) + f_3(y-\omega^2 x) + \frac{x^6 y^3}{120} + \frac{x^9}{10080}$

75. Which one of the following solutions of the partial differential equation $\frac{\partial^2 u}{\partial x \partial y} - u = 0$ can be obtained via separation of variables?

a. $u(x, y) = ce^{kx}\sin(ky)$

b. $u(x, y) = ce^{kx}e^{ky}$

c. $u(x, y) = ce^{kx}e^{y/k}$

d. $u(x, y) = cy\sin(kx)$

76. Which two out of the following four conditions must a differential equation satisfy so that the principle of superposition applies to its solutions?

(i) Constant coefficient

(ii) Homogeneous

(iii) Linear

(iv) Second order

a. (i) and (iii)

b. (ii) and (iii)

c. (i) and (ii)

d. (ii) and (iv)

77. The partial differential equation arising from the relation $u(x, y) = f(x + iy) + g(x - iy)$ is classified as

a. singular

b. hyperbolic

c. parabolic

d. elliptic

78. Which one of the following satisfies the partial differential equation

$$\frac{\partial^3 u}{\partial x^2 \partial y} = \cos(2x + 3y)?$$

a. $u(x, y) = -\frac{1}{12}\sin(3x + 2y) + xf_1(y) + f_2(y)f_3(x)$

b. $u(x, y) = -\frac{1}{12}\sin(2x + 3y) + yf_1(y) + f_2(y)f_3(x)$

c. $u(x, y) = -\frac{1}{12}\sin(2x - 3y) + xf_1(y) + f_2(y)f_3(x)$

d. $u(x, y) = -\frac{1}{12}\sin(2x + 3y) + xf_1(y) + f_2(y)f_3(x)$

79. The complete solution of a differential equation contains arbitrary constants

a. more than the order of the equation

b. Cannot be determined

c. equal to the order of the equation

d. less than the order of the equation

80. The equation

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$$

is classified as

a. elliptic

b. hyperbolic

c. parabolic

d. two-dimensional heat equation

81. In the nonlinear equation $x^2 - 2x - 3 = 0$

a. the sequence generated by $x_{i+1} = \frac{x_i^2 - 3}{2}$ converges to a root of the above nonlinear equation

b. the sequence generated by $x_{i+1} = \frac{3}{x_i - 2}$ does not converge to a root of the above nonlinear equation

c. the sequence generated by $x_{i+1} = \frac{x_i^2 - 3}{2}$ does not converge to a root of the above nonlinear equation

d. the sequence generated by $x_{i+1} = \sqrt{2x_i + 3}$ does not converge to a root of the above nonlinear equation

82. LU decomposition of a square matrix

a. is unique

b. is not unique

c. does not exist

d. may or may not be unique

83. The spectral radius of the following matrix

$$\begin{bmatrix} 0 & \frac{1}{3} & \frac{1}{4} \\ -\frac{1}{3} & 0 & \frac{1}{2} \\ -\frac{1}{4} & -\frac{1}{2} & 0 \end{bmatrix}$$

is

a. $\frac{5}{6}$

b. $\frac{7}{12}$

c. $\frac{3}{4}$

d. $\frac{1}{3}$

84. The truncation error in calculating $f'(2)$ for $f(x) = x^2$ by $f'(x) \approx \frac{f(x+h) - f(x)}{h}$ with $h = 0 \cdot 2$ is

a. $-0{\cdot}2$

b. $0{\cdot}2$

c. 4

d. $4{\cdot}2$

85. The value of $\int_3^{19} f(x)\,dx$ by using 2-segment Simpson's $\frac{1}{3}$rd rule is estimated to be $702{\cdot}039$. The estimate of the same integral using 4-segment Simpson's $\frac{1}{3}$rd rule most nearly is

a. $702{\cdot}039 + \frac{8}{3}[2f(7) - f(11) + 2f(15)]$

b. $\frac{702 \cdot 039}{2} + \frac{8}{3}[2f(7) - f(11) + 2f(15)]$

c. $702{\cdot}039 + \frac{8}{3}[2f(7) + 2f(15)]$

d. $\frac{702 \cdot 039}{2} + \frac{8}{3}[2f(7) + 2f(15)]$

86. The degree of precision of the quadrature formula

$$\int_{-1}^{1} f(x)\,dx = f\left(-\frac{\sqrt{3}}{3}\right) + f\left(\frac{\sqrt{3}}{3}\right)$$

is

a. 3

b. 2

c. 1

d. 5

87. If the trapezoidal rule applied to $\int_0^2 f(x)\,dx$ gives the value 5 and Simpson's rule gives the value 2, then $f(1)$ is

a. $\frac{1}{2}$

b. 1

c. 2

d. $\frac{1}{3}$

88. The following data of the velocity of a body is given as a function of time :

Time (in s)	4	7	10	15
Velocity (in m/s)	22	24	37	46

The best estimate of the distance in metres covered by the body from $t = 4$ to $t = 15$ using combined Simpson's $\frac{1}{3}$rd rule and the trapezoidal rule would be

a. 354·70

b. 362·50

c. 368

d. 378·80

89. The Lagrange's polynomial that passes through the 3 data points is given by

x	15	18	22
y	24	37	25

$$f_2(x) = L_0(x)(24) + L_1(x)(37) + L_2(x)(25)$$

The value of $L_1(x)$ at $x = 16$ is most nearly

a. –0·071430

b. 0·50000

c. 0·57143

d. 4·3333

90. If Taylor's method of order n is used to approximate the solution to $y'(x) = f(x, y(x))$, $a \le x \le b$, $y(a) = \alpha$ with step size h and if $y \in C^{n+1}[a, b]$, then the local truncation error is

a. $O(h^2)$

b. $O(h)$

c. $O(h^{n+1})$

d. $O(h^n)$

91. Let $C = \{(x, y, z) : 3 \le x \le 18, 2 \le y \le 18, 1 \le z \le 18\}$ be the sample space. Let $A = \{(x, y, z) : x + y + z = 18 \text{ and } x \ge 3, y \ge 2, z \ge 1\}$. Which one of the following is the probability of A?

a. $\frac{91}{16 \times 17 \times 18}$

b. $\frac{59}{16 \times 17 \times 18}$

c. $\frac{78}{16 \times 17 \times 18}$

d. $\frac{64}{16 \times 17 \times 18}$

92. How many dice should be thrown so that there is a better than an even chance of obtaining a six?

a. 2

b. 3

c. 4

d. None of the above

93. There is a 30 percent chance that it will rain on any particular day. Given that there is at least one rainy day. What is the probability that there are at least two rainy days?

a. 0·67

b. 0·91

c. 0·73

d. 0·81

94. If the probability distribution of a random variable X is $f(x) = k \sin \frac{\pi x}{5}$, $0 \le x \le 5$, then the value of k is

a. $\frac{\pi}{5}$

b. $\frac{\pi}{10}$

c. $\frac{\pi}{2}$

d. $\frac{2\pi}{5}$

95. If A and B are two events such that probabilities $P(A) = \frac{3}{4}$ and $P(B) = \frac{5}{8}$, then

a. $\frac{1}{4} \le P(A \cap B) \le \frac{1}{2}$

b. $\frac{3}{8} \le P(A \cap B) \le \frac{1}{2}$

c. $\frac{1}{4} \le P(A \cap B) \le \frac{3}{8}$

d. $\frac{3}{8} \le P(A \cap B) \le \frac{5}{8}$

96. Maximize $Z = x + 2y$

subject to

$$\begin{aligned} 2x + 3y &\le 18 \\ x + 4y &\le 18 \\ x \ge 0,\ y &\ge 0 \end{aligned}$$

If S denotes the set of all basic solutions to the above problem, then

a. S is empty

b. S is singleton

c. S is finite

d. S is uncountable

97. Consider the following LPP :

Maximize $Z = x + 2y$

subject to

$$\begin{aligned} 2x + y &\le 8 \\ 4x + 0y &\le 15 \\ x + 3y &\le 9 \\ x + y &\le k \\ x \ge 0,\ y &\ge 0 \end{aligned}$$

The value of k for which the constraint $x + y \le k$ is redundant, is

a. 2

b. 4

c. 6

d. 8

98. Consider the following LPP :

Maximize $Z = x_1 + 5x_2 + 3x_3$

subject to

$$\begin{aligned} 2x_1 - 3x_2 + 5x_3 &\leq 3 \\ 3x_1 + 2x_3 &\leq 5 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

Then the dual of this LPP

a. has a feasible solution but does not have a basic feasible solution

b. has a basic feasible solution

c. has an infinite number of feasible solutions

d. has no feasible solution

99. Consider the following LPP :

Maximize $Z = x + y$

subject to

$$\begin{aligned} x - 2y &\leq 10 \\ y - 2x &\leq 10 \\ x, y &\geq 0 \end{aligned}$$

Then

a. the LPP admits an optimal solution

b. the LPP is unbounded

c. the LPP admits no feasible solution

d. the LPP admits a unique feasible solution

100. Which one of the following statements is true?

a. An LPP can have a non-basic optimal solution

b. An LPP can have infinite many extreme points

c. An LPP can have exactly two different optimal solutions

d. A convex set cannot have infinite many extreme points

SPACE FOR ROUGH WORK

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