Sample Question Paper for MSc Applied Mathematics

Details of Syllabus

Format of the Entrance Test Paper

The duration of the Entrance Test will be 2 hours and the question paper will consist of 50 multiple choice questions.

Calculus and Analysis: Limit, continuity, uniform continuity and differentiability; Bolzano Weierstrass theorem; mean value theorems; tangents and normal; maxima and minima; theorems of integral calculus; sequences and series of functions; uniform convergence; power series; Riemann sums; Riemann integration; definite and improper integrals; partial derivatives and Leibnitz theorem; total derivatives; Fourier series; functions of several variables; multiple integrals; line; surface and volume integrals; theorems of Green; Stokes and Gauss; curl; divergence and gradient of vectors.

Algebra: Basic theory of matrices and determinants; groups and their elementary properties; subgroups, normal subgroups, cyclic groups, permutation groups; Lagrange's theorem; quotient groups; homomorphism of groups; isomorphism and correspondence theorems; rings; integral domains and fields; ring homomorphism and ideals; vector space, vector subspace, linear independence of vectors, basis and dimension of a vector space.

Differential equations: General and particular solutions of ordinary differential equations (ODEs); formation of ODE; order, degree and classification of ODEs; integrating factor and linear equations; first order and higher degree linear differential equations with constant coefficients; variation of parameter; equation reducible to linear form; linear and quasi-linear first order partial differential equations (PDEs); Lagrange and Charpits methods for first order PDE; general solutions of higher order PDEs with constant coefficients.

Numerical Analysis: Computer arithmetic; machine computation; bisection, secant; Newton-Raphson and fixed point iteration methods for algebraic and transcendental equations; systems of linear equations: Gauss elimination, LU decomposition, Gauss Jacobi and Gauss Siedal methods, condition number; Finite difference operators; Newton and Lagrange interpolation; least square approximation; numerical differentiation; Trapezoidal and Simpsons integration methods.

Probability and Statistics: Mean, median, mode and standard deviation; conditional probability; independent events; total probability and Baye’s theorem; random variables; expectation, moments generating functions; density and distribution functions, conditional expectation.

Linear Programming: Linear programming problem and its formulation; graphical method, simplex method, artificial starting solution, sensitivity analysis, duality and post-optimality analysis.

Negative Marks for Wrong Answers: If the answer given to any of the Multiple Choice Questions is wrong, 1/4th of the marks assigned to that question will be deducted.

Calculators will not be allowed. However, Log Tables may be used.

- This is only a sample paper and only meant to be indicative of the type of questions that will be asked.

1. The graph of $y = 5x^4 - x^5$ has a point of inflection at
   a. (0,0) only
   b. (3,162) only
   c. (4,256) only
   d. (0,0) and (3,162)

2. If $y = \tan u$, $u = v - \frac{1}{v}$ and $v = \ln x$, what is the value of $\frac{dy}{dx}$ at $x = e$?
   a. 0
   b. $\frac{2}{e}$
   c. $\frac{1}{e}$
   d. 1

3. If a function ***f*** is defined as follows:

$$f(x) = \begin{cases} 1 + x, & \text{if } x \le 2 \\ 5 - x, & \text{if } x > 2 \end{cases}$$

Then which one of the following is true?
   a. f is continuous but not differentiable at $x = 2$
   b. f is differentiable at every point of R
   c. f is neither continuous nor differentiable at $x = 2$
   d. f is differentiable at $x = 2$ but is not continuous at $x = 2$

4. Let $f(x)=\left|\sin x-\frac{1}{2}\right|$. The maximum value attained by f is
   a. $\frac{1}{2}$
   b. 1
   c. $\frac{3}{2}$
   d. $\frac{\pi}{2}$
5. The integral $\int_0^1 \frac{\ln(1+x)}{x} dx$ converges to
   a. $\frac{\pi^2}{12}$
   b. $\frac{\pi^2}{6}$
   c. $\frac{\pi}{12}$
   d. $\frac{\pi}{6}$

6. The value of the integral $\int_{1/2}^{1} \frac{dx}{x^4\sqrt{1-x^2}}$ is
   a. $2/\sqrt{3}$
   b. $2\sqrt{3}$
   c. $3/\sqrt{3}$
   d. $3\sqrt{3}$
7. The integral $\int_1^\infty \frac{x^{m-1}}{1+x} dx$ converges if
   a. $m > 3$
   b. $m > 2$
   c. $m < 1$
   d. For all $m \in R$
8. Consider the function f defined as follows

$$f(x) = \begin{cases} 0 & for\ \ -\pi < x < 0 \\ \pi & for\ \ 0 < x < \pi \end{cases}$$

   and suppose Fourier series of f is $\frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$, then

   a. $a_0 = 0, a_n = 0, b_n = n, n \geq 1$
   b. $a_0 = 0, a_n = 0, b_n = 2/n, n \geq 1$
   c. $a_0 = \pi, a_n = \frac{1-\cos n\pi}{n}, b_n = 0, n \geq 1$
   d. $a_0 = \pi, a_n = 0, b_n = \frac{1-\cos n\pi}{n}, n \geq 1.$
9. The integral $\int_R xy(x^2+y^2)dxdy$ over the rectangular region $R: 0 \leq x \leq a, 0 \leq y \leq b$ is
   a. $\frac{1}{8}a^2b^2(a^2+b^2)$

b. $\frac{1}{8}a^2b^3(a^2+b^2)$
c. $\frac{1}{8}a^3b^2(a^2+b^2)$
d. $\frac{1}{8}a^3b^3(a^2+b^2)$

10. If $f(x)=2+|x-3|$ for all x, then the value of the derivative $f'(x)$ at x = 3 is

a. -1
b. 0
c. 1
d. does not exist

11. The function $f(x)=\sin x$ is increasing in the interval

a. $[0,\pi]$
b. $[0,\frac{\pi}{2}]$
c. $[\frac{\pi}{4},\frac{3\pi}{4}]$
d. $[\frac{\pi}{2},\pi]$

12. The value of 'c' in Rolle's theorem for the function $f(x)=e^x\sin x$ in $[0,\pi]$ is given by

a. $c=\frac{3\pi}{4}$
b. $c=\frac{\pi}{4}$
c. $c=\frac{\pi}{2}$
d. $c=\frac{5\pi}{6}$

13. The line integral $\int_\Gamma (\cos x\sin y - xy)dx + (\sin x\cos y)dy$, where Γ is the circle $x^2+y^2=1$ in xy-plane described in anticlockwise direction, is
a. -2

b. -1

c. 0

d. 1

14. The surface integral $\int \int_S (x^3dydz + y^3dzdx + z^3dxdy)$ over the sphere $x^2 + y^2 + z^2 = a^2$ is

a. $\frac{12}{5}\pi a^4$

b. $\frac{12}{7}\pi a^5$

c. $\frac{12}{5}\pi a^5$

d. $\frac{12}{7}\pi a^4$

15. Consider the function f defined as follows

$$f(x) = \begin{cases} 1, & 0 \leq x < \frac{\pi}{3} \\ 0, & \frac{\pi}{3} \leq x \leq \frac{2\pi}{3} \\ -1, & \frac{2\pi}{3} < x \leq \pi. \end{cases}$$

If f has the Fourier series $\frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$, then

a. $a_0 = 0, a_n = 0, b_n = \cos n\,, n \geq 1$

b. $a_0 = 0, a_n = 0, b_n = 2/n, n \geq 1$

c. $a_0 = 0, a_n = 0, b_n = \sum_{n=1}^{\infty} \frac{2}{n\pi}\left(\sin\frac{n\pi}{3} + \cos\frac{n\pi}{3}\right), n \geq 1$

d. $a_0 = 0, a_n = \sum_{n=1}^{\infty} \frac{2}{n\pi}\left(\sin\frac{n\pi}{3} + \cos\frac{n\pi}{3}\right), b_n = 0, n \geq 1.$

16. If x be an element of a group of order 8, then the order of x^3 is

a. 24

b. 11

c. 3

d. 8

17. In the group$G = \{2,4,6,8\}$under multiplication modulo 10, the identity element is

a. 2

b. 4

c. 6

d. 8

18. Which of the following statements is true?

a. The identity permutation is both odd as well as even

b. The product of two cycles is a cycle

c. Two disjoint cycles commute
d. The inverse of (1346) is (6134)

19. The number of left cosets of subgroup$H = \{1,11\}$ in $U(30)$ is
   a. 7
   b. 4
   c. 5
   d. 8
20. If G is a group of order 36 and H, K are subgroups of order 4 and 18, respectively, then the index of H∩ K in G is
   a. 2
   b. 4
   c. 6
   d. 18

21. Let G be a non abelian group of order p^3 , where p is prime. Then order of Z(G) is
   a. 1
   b. p
   c. p^3
   d. 1 or p

22. If every subgroup of a group is normal, then the group is
   a. Abelian
   b. Cylic
   c. Simple
   d. None of the above

23. The number of homomorphism from Z_{24} onto S_3 is
   a. 0
   b. 1
   c. 2
   d. 3

24. Consider the initial value problem (IVP) $y' = y^2,\ \ y(0) = 3$. Then $y(x) = \frac{1}{(1-x)}$

   a. satisfies the IVP

   b. satisfies the differential equation but not the initial condition

c. satisfies the initial condition but not the differential equation

d. does not satisfy either the differential equation or the initial condition

25. The integrating factor for the differential equation $xy' + 2y = \cos(x)$is

a. e^{2x}

b. x^2

c. $\ln x$

d. $\sin\ x$

26. The over damped motion of a particle in potential $V(x)$ satisfies the first order differential equation

$$\gamma \frac{dx}{dt} = -V'(x),$$

where γ is the constant coefficient of friction. The solution of this equation is

a. $V(x) + \frac{\gamma x^2}{2} = constant$

b. $V(x) + \gamma x = constant$

c. $\frac{t}{\gamma} + \frac{\int dx}{V\prime(x)} = 0$

d. None of the above

27. The equation $\frac{\partial z}{\partial x} e^y = \frac{\partial z}{\partial y} e^x$ has the general solution

a. z = a e^x + b e^x

b. z = e^x + e^y

c. z = a (e^x + e^y) + b

d. None of the above

28. The Partial differential equation $U_t = U_{xx}$ is

a. Laplace equation

b. Wave equation

c. Two dimensional heat conduction equation

d. One dimensional heat conduction equation

29. The general solution of the differential equation$2yzp + zxq = 3xy$ is

a. $\varphi(x^2 - 2y^2\,, 3y^2 - z^2) = 0$
b. $\varphi(x^2 - 2y^2\,, 3y^2 - z) = 0$
c. $\varphi(x - 2y^2\,, 3y - z) = 0$
d. $\varphi(x^2 - 2y\,, 3y^2 - z^2) = 0$

30. If f(x) is continuous and possesses continuous derivative of order n in an interval containing x = a, then

$$f(x) = f(a) + (x-a)f'(a) + \cdots\cdots + \frac{(x-a)^{n-1}}{(n-1)!} f^{(n-1)}(a) + R_n(x).$$

The expression of remainder term $R_n(x)$ is given by

(a) $R_n(x) = \frac{(x-a)^n}{n!} f^{(n+1)}(\zeta), a < \zeta < x$
(b) $R_n(x) = \frac{(x-a)^n}{(n+1)!} f^{(n)}(\zeta), a < \zeta < x$
(c) $R_n(x) = \frac{(x-a)^n}{n!} f^{(n)}(\zeta), a < \zeta < x$
(d) $R_n(x) = \frac{(x+a)^n}{n!} f^{(n)}(\zeta), a < \zeta < x$

31. The order of convergence of Newton-Raphson method is
a. 1
b. 2
c. 3
d. 1.5

32. If for a real continuous function ϕ, $\phi(a)\phi(b) < 0$, then in the interval $[a, b]$, ϕ has
a. only one root
b. no root
c. an undeterminable number of roots
d. at least one root

33. Given six data pairs, a unique polynomial of degree α passes through the six data points. The value of α is given by
a. $\alpha \leq 5$
b. $\alpha = 6$
c. $\alpha = 1$
d. $\alpha > 5$

34. The next iterative value of the root of $x^2 - 4 = 0$ using Secant method, if the initial guesses are 3 and 4, is
    a. 5.5
    b. 5.7143
    c. 2.5
    d. 2.2857

35. Errors may occur in performing numerical computations on the computer due to
    a. power fluctuation
    b. operator fatigue
    c. rounding errors
    d. all of the above

36. The highest order of polynomial integrand for which Simpson's 1/3 rule of integration gives exact value is
    a. first
    b. third
    c. second
    d. fourth

37. The Jacobi iteration scheme to solve the system of equations

$$\begin{bmatrix} 4 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 3 \end{bmatrix}$$

is given by

a. $$\begin{bmatrix} x^{(k+1)} \\ y^{(k+1)} \\ z^{(k+1)} \end{bmatrix} = \begin{bmatrix} 0 & 1/4 & 0 \\ 1/4 & 0 & 1/4 \\ 0 & 1/4 & 0 \end{bmatrix} \begin{bmatrix} x^{(k)} \\ y^{(k)} \\ z^{(k)} \end{bmatrix} + \begin{bmatrix} 3/4 \\ 1/2 \\ 3/4 \end{bmatrix}$$

b. $$\begin{bmatrix} x^{(k+1)} \\ y^{(k+1)} \\ z^{(k+1)} \end{bmatrix} = \begin{bmatrix} 1 & 1/4 & 0 \\ 1/4 & 0 & 1/4 \\ 0 & 1/4 & 0 \end{bmatrix} \begin{bmatrix} x^{(k)} \\ y^{(k)} \\ z^{(k)} \end{bmatrix} + \begin{bmatrix} 3/4 \\ 1/2 \\ 3/4 \end{bmatrix}$$

c. $$\begin{bmatrix} x^{(k+1)} \\ y^{(k+1)} \\ z^{(k+1)} \end{bmatrix} = \begin{bmatrix} 0 & 4 & 0 \\ 1/4 & 0 & 1/4 \\ 0 & 1/4 & 0 \end{bmatrix} \begin{bmatrix} x^{(k)} \\ y^{(k)} \\ z^{(k)} \end{bmatrix} + \begin{bmatrix} 3/4 \\ 1/2 \\ 3/4 \end{bmatrix}$$

d. $\begin{bmatrix} x^{(k+1)} \\ y^{(k+1)} \\ z^{(k+1)} \end{bmatrix} = \begin{bmatrix} 0 & 1/4 & 0 \\ 1/4 & 0 & 1/4 \\ 0 & 1/4 & 1 \end{bmatrix} \begin{bmatrix} x^{(k)} \\ y^{(k)} \\ z^{(k)} \end{bmatrix} + \begin{bmatrix} 3/4 \\ 1/2 \\ 3/4 \end{bmatrix}$

38. If the difference of mode and median of a data is 50, then the difference of median and mean is
    a. 25
    b. 21
    c. 27
    d. 30

39. The mean of the 16 observations is 16. If one observation 16 is deleted and three observations 5, 5, 6 are included, then the mean of new observations is
    a. 16
    b. 14.22
    c. 15.5
    d. 13.5

40. A sum of money is rounded off to the nearest rupee. The probability that round off error is at least ten paise is
    a. $\frac{81}{101}$
    b. $\frac{80}{100}$
    c. $\frac{81}{100}$
    d. $\frac{19}{101}$

41. An elevator starts with 4 passengers and stops at 4 floors. The probability that no two passengers alight at the same floor is
    a. $\frac{12}{128}$
    b. $\frac{1}{32}$
    c. $\frac{3}{128}$
    d. $\frac{3}{64}$

42. Afghanistan plays two matches each with India and Pakistan. In any match the probabilities of Afghanistan getting points 0, 1 and 2 are 0.45, 0.05 and 0.50, respectively. Assuming that the outcomes are independent, the probability of Afghanistan getting at least 7 points is

a. 0.8750
b. 0.0875
c. 0.0625
d. 0.0250

43. A wire of length lunit is cut into three pieces. What is the probability that the three pieces form a triangle?

a. $1/2$
b. $2/3$
c. $1/4$
d. $1/3$

44. If the range of a random variable X is 0, 1, 2, 3, 4⋯with $P(X = k) = \frac{k+1}{3^k} a$ for $k \geq 0$, then $a =$

a. $\frac{8}{27}$
b. $\frac{16}{81}$
c. $\frac{4}{9}$
d. $\frac{2}{3}$

45. In less than or equal to constraint equations, the variable which is used to balance both sides of equations is classified as

a. Artificial variable
b. Slack variable
c. Surplus variable
d. None of the above

46. If in a standard form LPP, we have a linear system with n nonnegative variables and m equations, then the maximum number of basic solutions is given by

a. $\frac{n!}{m!(n-m)!}$

b. $\frac{m!}{n!(m-n)!}$
c. n!
d. m!

47. In the simplex method, slack, surplus and artificial variables are restricted to be

a. negative
b. non-negative
c. unrestricted
d. none of the above

48. According to the algebra of the simplex method, slack variables are assigned zero coefficients because
a. there is no contribution in objective function
b. there is high contribution in objective function
c. slack variables should be zero in the optimal table
d. it makes calculations easy

49. Choose the most correct of the following statements relating to primal-dual linear programming problems:

a. Shadow prices of resources in the primal are optimal values of dual variables.
b. The optimal values of the objective functions of the primal and dual are the same.
c. If the primal problem has unbounded solution, the dual problem would have infeasibility.
d. All of the above.

50. Apply linear programming to this problem: David and Harry operate a discount jewellery store. They want to determine the best mix of customers to serve each day. There are two types of customers for their store, retail (R) and wholesale (W). The cost to serve a retail customer is Rs. 70 and the cost to serve a wholesale customer is Rs. 89. The average profit from either kind of customer is the same. To meet headquarters' expectations, they must serve at least 8 retail customers and 12 wholesale customers daily. In addition, in order to cover their salaries, they must at least serve 30 customers each day. Which of the following is one of the constraints for this model?

a. $1\,R + 1\,W \leq 8$
b. $1\,R + 1\,W \geq 30$
c. $8\,R + 12\,W \geq 30$
d. $1\,R \geq 12$