14

QUESTION PAPER SERIES CODE
A

Centre of Examination : ____________________

Roll No. : ____________________

Name of Candidate : ____________________

S A U

Entrance Test for Ph.D. (Applied Mathematics)

[2013]

Time : 3 hours | *Maximum Marks* : 70

INSTRUCTIONS FOR CANDIDATES

Candidates must carefully read the following instructions before attempting the Question Paper :

(i) Write your Name and Roll Number in the space provided for the purpose on the top of this Question Paper and in the OMR/Answer Sheet.

(ii) This Question Paper has Two Parts : Part—A and Part—B.

(iii) Part—A has 30 questions (Objective-type) of **1** mark each. All questions are compulsory.

(iv) Part—B has 40 questions (Objective-type) of **1** mark each. All questions are compulsory.

(v) Symbols have their usual meanings.

(vi) **Please darken the appropriate Circle of 'Question Paper Series Code' on the OMR/Answer Sheet in the space provided.**

(vii) Questions for both the Parts should be answered on OMR/Answer Sheet.

(viii) Answers written by the candidates inside the Question Paper will **NOT** be evaluated.

(ix) Calculators and Log Tables may be used.

(x) Pages at the end have been provided for Rough Work.

(xi) **Return the Question Paper and the OMR/Answer Sheet** to the Invigilator at the end of the Entrance Test.

(xii) **DO NOT FOLD THE OMR/ANSWER SHEET.**

INSTRUCTIONS FOR MARKING ANSWERS IN THE 'OMR SHEET'

1. **Please ensure that you have darkened the appropriate Circle of 'Question Paper Series Code' on the OMR Sheet in the space provided.**
2. Use only Blue/Black Ballpoint Pen to darken the Circle. Do not use Pencil to darken the Circle for Final Answer.
3. Please darken the whole Circle.
4. Darken ONLY ONE CIRCLE for each question as shown below in the example.

Example :

Wrong	Wrong	Wrong	Wrong	Correct
● ⓑ ⓒ ●	~~ⓐ~~ ⓑ ⓒ ⓓ	~~ⓐ~~ ⓑ ⓒ ~~ⓓ~~	⦿ ⓑ ⓒ ●	ⓐ ⓑ ⓒ ●

5. Once marked, no change in the answer is allowed.
6. Please do not make any stray marks on the OMR Sheet.
7. Please do not do any rough work on the OMR Sheet.
8. Mark your answer only in the appropriate Circle against the number corresponding to the question.
9. There will be no negative marking in evaluation.

PART—A

1. The series

$$1-\frac{1}{2}+\frac{1}{3}+\frac{1}{4}-\frac{1}{5}+\frac{1}{6}+\frac{1}{7}-\frac{1}{8}+\frac{1}{9}+\frac{1}{10}-\frac{1}{11}+\cdots$$

is

(a) not convergent

(b) convergent but not absolutely convergent

(c) absolutely convergent

(d) absolutely convergent but not convergent

2. The maximum value of $f(x)=x^{1/x}$ defined on $(0, \infty)$ is attained at

(a) $x=1$

(b) $x=e^2$

(c) $x=e$

(d) $x=1/e$

3. Let $a_1, a_2, \cdots, a_n, \cdots$ be a sequence of positive real numbers such that

$$\sum_{n=1}^{\infty} a_n^2$$

converges. Which of the following necessarily holds?

(a) $\sum_{n=1}^{\infty} a_n$ converges

(b) $\sum_{n=1}^{\infty} \frac{a_n}{n}$ converges

(c) $\sum_{n=1}^{\infty} n a_n$ converges

(d) None of the above

4. The function

$$f(x) = \begin{cases} x, & \text{when } x \text{ is rational} \\ -x, & \text{when } x \text{ is irrational} \end{cases}$$

satisfies that

(a) it is discontinuous at every point

(b) it is continuous at every point

(c) it is continuous only at $x = 0$

(d) it is discontinuous only at $x = 0$

5. The inverse of the element 5 in the group $U(8) = \{x \in \mathbb{Z}_8 : (x, 8) = 1\}$ is

(a) 1

(b) 2

(c) 5

(d) 7

6. The order of the permutation $(1\ 12\ 8\ 10\ 4)(2\ 13)(5\ 11\ 7)(6\ 9) \in S_{13}$ is

(a) 30

(b) 15

(c) 10

(d) 5

7. If $H = \langle x \rangle$ and $o(x) = 10$, then the number of generators of H is

(a) 1

(b) 2

(c) 4

(d) 8

8. Let $G = \mathbb{Z}$ under usual addition and $H = \mathbb{Z}_n$ under addition modulo n be two groups. Let $f: G \to H$ be defined by $f(x) = \bar{x}\ \forall\, x \in \mathbb{Z}$. Then

(a) f is a homomorphism, one-one but not onto

(b) f is an isomorphism

(c) f is a homomorphism, onto but not one-one

(d) f is not a homomorphism

9. The general solution of the differential equation

$$\frac{dy}{dx} - \frac{dx}{dy} = \frac{x}{y} - \frac{y}{x}$$

is

(a) $x^2 + y^2 = c$

(b) $(xy - c)(y^2 + x^2 - c) = 0$

(c) $x + y = 0$

(d) $(xy - c)(y^2 - x^2 - c) = 0$

10. The general solution of the linear differential equation

$$\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0$$

is

(a) $c_1 e^{-x} + c_2 \cos(4x)$

(b) $c_1 e^{-x} + c_2 \sin(x) + c_3 \cos(x)$

(c) $c_1 e^{-x} + c_2 \sin(2x) + c_3 \cos(2x)$

(d) $c_1 e^{x} + c_2 \sin(3x) + c_3 \cos(3x)$

11. The general solution of the partial differential equation

$$(x^2 - y^2 - z^2)\frac{\partial z}{\partial x} + 2xy\frac{\partial z}{\partial y} = 2xz$$

is given by

(a) $x^2 + y^2 + z^2 = zf\left(\frac{y}{z}\right)$

(b) $x^2 + y^2 = zf\left(\frac{y}{z}\right)$

(c) $x^2 + y^2 + z^2 = f\left(\frac{y}{z}\right)$

(d) $z^2 = f\left(\frac{y}{z}\right)$

12. The solution of the partial differential equation

$$\frac{\partial z}{\partial x}\left(\left(\frac{\partial z}{\partial y}\right)^2 + 1\right) + (b - z)\frac{\partial z}{\partial y} = 0$$

is

(a) $2\sqrt{a(z-b)-1} = x + ay + b$

(b) $2\sqrt{a(z-b)-1} = x - ay + b$

(c) $2z = x + ay + b$

(d) $2\sqrt{z+b} = x - y + b$

13. For an equation like $x^2 = 0$, a root exists at $x = 0$. The bisection method cannot be adopted to solve this equation in spite of the root existing at $x = 0$, because the function $f(x) = x^2$

(a) is a polynomial

(b) has repeated roots at $x = 0$

(c) is always non-negative

(d) has a slope equal to zero at $x = 0$

14. The unique solution for the system of linear equations

$$\begin{bmatrix} 2 & 3 & 0 & 0 \\ 2 & 4 & 1 & 0 \\ 0 & 2 & 6 & A \\ 0 & 0 & 4 & B \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 4 \\ 0 \end{bmatrix}$$

would not exist, if

(a) $A - B = 0$

(b) $A + B = 0$

(c) $A + 2B = 0$

(d) $2A + B = 0$

15. The two-segment trapezoidal rule of integration is exact for integrating polynomials of what degree?

(a) First

(b) Second

(c) Third

(d) Fourth

16. The line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and the plane $2x - 4y + 2z = 3$ meet in

(a) no point

(b) only one point

(c) finitely many points

(d) infinitely many points

17. The direction cosines of x-axis are

(a) (0, 0, 1)

(b) (1, 0, 0)

(c) (0, 1, 0)

(d) (0, 1, 1)

18. The maximum value of the directional derivative of $\phi = x^2yz$ at the point (1, 4, 1) is

(a) 81 (b) 9

(c) 3 (d) 0

19. For any closed surface S, the value of $\iint_S \text{curl}\, \vec{F} \cdot \hat{n}\, ds$ is

(a) unity

(b) variable

(c) zero

(d) Any of the above

20. If M and N are continuous functions of x and y in the region R with boundary C, then Green's theorem in the plane states that the value of $\oint_C (Mdx + Ndy)$ is

(a) $\iint_R \left(\frac{\partial M}{\partial x} - \frac{\partial N}{\partial y}\right) dxdy$

(b) $\iint_R \left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) dxdy$

(c) $\iint_R \left(\frac{\partial N}{\partial y} - \frac{\partial M}{\partial x}\right) dxdy$

(d) $\iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dxdy$

21. A ball is thrown at an angle of 45° from the horizontal with a speed of $V_A = 9\cdot81\sqrt{2}$ m s^{-1}. Then the maximum height it attains is

(a) 9·81 metres

(b) 4·905 metres

(c) 14·72 metres

(d) 19·62 metres

22. A $2\cdot0 \times 10^3$ kg car travels at a constant speed of $12\cdot0$ m s^{-1} around a circular curve of radius 30·0 metres. The magnitude of the centripetal acceleration of the car as it goes around the curve is

(a) 0·40 m s^{-2}

(b) 4·8 m s^{-2}

(c) 800 m s^{-2}

(d) 9600 m s^{-2}

23. A small block slides from rest from the top of a frictionless sphere of radius R. How far below the top x does it lose contact with the sphere?

(a) $R/2$

(b) $R/3$

(c) $R/4$

(d) $3R/4$

24. Which of the following statements is correct?

(a) Every LPP has at least one optimal solution

(b) Every LPP has a unique optimal solution

(c) If an LPP has two optimal solutions, then it has infinitely many solutions

(d) None of the above

25. Which of the following sets is not convex?

(a) $\{(x, y) : x + y \leq 1\}$

(b) $\{(x, y) : x^2 + y^2 \leq 1\}$

(c) $\{(x, y) : x + y \geq 1\}$

(d) $\{(x, y) : x^2 + y^2 \geq 1\}$

26. The maximum value of $f = 4x + 3y$, subject to constraints $x \geq 0$, $y \geq 0$, $2x + 3y \leq 18$, $x + y \geq 10$, is

(a) 35

(b) 36

(c) 34

(d) None of the above

27. The number of telephone calls arriving at a switchboard during any 10-minute period is known to be a Poisson random variable X with $\lambda = 2$. Then the probability that more than 3 calls will arrive during any 10-minute period is

(a) 0·135

(b) 0·143

(c) 0·159

(d) 0·174

28. A random variable R has probability density $f_R(x) = ke^{-\lambda x}$, $\lambda > 0$, $-\infty < x < \infty$, where k is a constant. Then the variance of R is given by

(a) 0

(b) $2/\lambda$

(c) $2/\lambda^2$

(d) $4/\lambda^2$

29. A system consisting of n separate components is first placed in series composition and then in parallel way. Assume that the components fail independently and the probability of failure of component i is p_i, $i = 1, 2, \cdots, n$. The probability for both the series and parallel compositions of systems works respectively is

(a) $(1-p_1)(1-p_2)\cdots(1-p_n)$, $1-p_1p_2\cdots p_n$

(b) $(1-p_1)(1-p_2)\cdots(1-p_n)$, $p_1p_2\cdots p_n$

(c) $1-p_1p_2\cdots p_n$, $1-(1-p_1)(1-p_2)\cdots(1-p_n)$

(d) $1-(1-p_1)(1-p_2)\cdots(1-p_n)$, $p_1p_2\cdots p_n$

30. Let R_1 and R_2 be independent. Assume that R_1 has the binomial distribution with parameters n and p, and R_2 has the binomial distribution with parameters m and p. Then the probability of the event is

(a) $\dfrac{{}^nC_{k-j}\,{}^mC_j}{\sum_{i=0}^{k} {}^nC_i\,{}^mC_{k-i}}$

(b) $\dfrac{{}^nC_j\,{}^mC_{k-j}}{{}^{m+n}C_k}$

(c) $\dfrac{{}^nC_j\,{}^mC_{k-j}}{{}^{m+n}C_k}\,p^k$

(d) $\dfrac{{}^nC_{k-j}\,{}^mC_j}{\sum_{i=0}^{k} {}^nC_i\,{}^mC_{k-i}}\,p^k$

PART—B

31. The range and kernel of $T : R^3 \to R^3$ defined by $T(x, y, z) = (x+z, x+y+2z, 2x+y+3z)$, where R, $\text{Im}(T)$ and $\ker(T)$ denote the set of real numbers, range and kernel of T respectively are

(a) $\text{Im}(T) = \{x(1, 0, 1) + y(0, 1, 1) : x, y \in R\}$, $\ker(T) = \{\alpha(1, 1, -1) : \alpha \in R\}$

(b) $\text{Im}(T) = \{x(1, 0, -1) + y(0, 1, 1) : x, y \in R\}$, $\ker(T) = \{\alpha(-1, 1, -1) : \alpha \in R\}$

(c) $\text{Im}(T) = \{x(1, 0, 1) + y(0, -1, -1) : x, y \in R\}$, $\ker(T) = \{\alpha(1, 1, -1) : \alpha \in R\}$

(d) $\text{Im}(T) = \{x(-1, 0, -1) + y(0, 1, 1) : x, y \in R\}$, $\ker(T) = \{\alpha(1, 1, -1) : \alpha \in R\}$

32. Let S_n and K_n be vector spaces of symmetric and skew-symmetric matrices of order $n \times n$ over the field of real numbers. Then

(a) dimension of $S_n = \frac{n(n+1)}{2}$, dimension of $K_n = \frac{n(n-1)}{2}$

(b) dimension of $S_n = \frac{n(n-1)}{2}$, dimension of $K_n = \frac{n(n+1)}{2}$

(c) dimension of $S_n = n(n+1)$, dimension of $K_n = n(n-1)$

(d) dimension of $S_n = n(n-1)$, dimension of $K_n = n(n+1)$

33. Consider the system of linear equations $x+2y+z=4$, $2x+y+2z=5$, $x-y+z=1$. Then it has

(a) a unique solution at $x=1$, $y=1$, $z=1$

(b) only two solutions $(x=1, y=1, z=1)$ and $(x=2, y=1, z=0)$

(c) infinite number of solutions

(d) no solution

34. The characteristic equation of a (3×3) matrix P is defined as $|\lambda I - P| = \lambda^3 + \lambda^2 + 2\lambda + 1 = 0$. If I denotes the identity matrix, then the inverse of P will be

(a) $(P^2 + P + 2I)$

(b) $(P^2 + P + I)$

(c) $-(P^2 + P + I)$

(d) $-(P^2 + P + 2I)$

35. Let A be a (2×2) matrix with elements $a_{11} = -1$, $a_{12} = a_{21} = a_{22} = 1$. Then the eigenvalues of the matrix A^{19} are

(a) 1024 and -1024

(b) $1024\sqrt{2}$ and $-1024\sqrt{2}$

(c) $4\sqrt{2}$ and $-4\sqrt{2}$

(d) $512\sqrt{2}$ and $-512\sqrt{2}$

36. $f(z) = \bar{z}$, $z \in \mathbb{C}$ satisfies which of the following?

(a) It is analytic everywhere

(b) It is analytic everywhere except at $z = 0$

(c) It is analytic nowhere except at $z = 0$

(d) It is analytic nowhere

37. The function

$$f(z) = \begin{cases} \dfrac{z - \sin z}{z^3}, & z \neq 0 \\ 0, & z = 0 \end{cases}$$

has

(a) a removable singularity at $z = 0$

(b) a pole of order 2 at $z = 0$

(c) a simple pole at $z = 0$

(d) an essential singularity at $z = 0$

38. Let $C = \{z : |z| = \frac{3}{2}\}$. Then the value of the integral

$$\int_C \frac{z^2 + 5z + 2}{(z-1)(z-2)(z-3)} dz$$

is

(a) $4\pi i$

(b) $6\pi i$

(c) $8\pi i$

(d) $10\pi i$

39. Let C be the circle centered at origin and with radius 1. Then the value of the integral

$$\int_C \frac{e^z}{z^3}\, dz$$

is

(a) $e^{\pi i}$ (b) πi

(c) $-\pi i$ (d) $e^{-\pi i}$

40. If $f(z) = 4z^3 - 3iz^2 + iz - 9$, then $f(z)$ satisfies which of the following?

(a) It has no zeros in $|z| < 1$

(b) It has only one zero in $|z| < 2$

(c) It has only two zeros in $|z| < 3$

(d) It has all the zeros in $|z| > 3$

41. Let $f : [0, 1] \rightarrow R$ be a function such that

$$f(x) = \begin{cases} 0, & \text{when } x \text{ is rational} \\ 1, & \text{when } x \text{ is irrational} \end{cases}$$

Which of the following is true?

(a) f is Riemann integrable over $[0, 1]$

(b) f is Lebesgue integrable over $[0, 1]$

(c) f is both Riemann as well as Lebesgue integrable over $[0, 1]$

(d) f is neither Riemann nor Lebesgue integrable over $[0, 1]$

42. Let X be the set of all irrational numbers with discrete metric. Then which of the following is true?

(a) X is complete

(b) X is compact

(c) X is connected

(d) X is bounded

43. Which of the following is not a metric on R?

(a) $d(x, y) = \min\{1, |x-y|\}$

(b) $d(x, y) = |x-y|/|1+x-y|$

(c) $d(x, y) = \sin|x-y|$

(d) $d(x, y) = |x-y|$

44. The number of fixed points of the mapping $T:(0, 1) \to (0, 1)$ defined by $T(x) = \frac{1}{x}$ is ____.

(a) 0 (b) 1

(c) 2 (d) 4

45. The dual space of ℓ^1 is

(a) ℓ^1 (b) ℓ^2

(c) ℓ^3 (d) ℓ^∞

46. Consider the set $X = \{0, 1, 2, 3, 4, 5, 6, 7\}$ under addition and multiplication modulo 8, then

(a) X is an integral domain

(b) X is a field

(c) X is a commutative ring

(d) X is a non-commutative ring

47. The polynomial $f(x) = 2x^2 + 4$ is

(a) irreducible over $\mathbb{Q}$

(b) reducible over $\mathbb{Z}$

(c) reducible over $\mathbb{R}$

(d) irreducible over $\mathbb{C}$

48. The splitting field of the polynomial $x^4 + 4$ over $\mathbb{Q}$ is

(a) $\mathbb{Q}(\sqrt{2})$ (b) $\mathbb{Q}$

(c) $\mathbb{Q}(i)$ (d) $\mathbb{Q}(\sqrt{2}, i)$

49. Let G be a group of order p^2, where p is a prime. Then

(a) G is non-Abelian

(b) G is Abelian

(c) G is cyclic

(d) G is non-cyclic

50. Let G be a group of order 21. Then which is not correct?

(a) G has a Sylow 7 subgroup

(b) Sylow 7 subgroup of G is normal

(c) G has 3 Sylow 3 subgroups

(d) G has 1 or 7 Sylow 3 subgroups

51. The solution of the first-order ordinary differential equation

$$\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$$

is

(a) $e^{-y} = e^{-x} + \frac{x^2}{2} + c$

(b) $e^{y} = e^{x} - \frac{x^3}{3} + c$

(c) $e^{y} = e^{-x} + x + c$

(d) $e^{y} = e^{x} + \frac{x^3}{3} + c$

52. The solution of the differential equation $\frac{dx}{yz} = \frac{dy}{zx} = \frac{dz}{xy}$ is

(a) $x^2 - y^2 = c_1,\ x^2 - z^2 = c_2$

(b) $x^2 + y^2 = c_1,\ x^2 + z^2 = c_2$

(c) $x - y = c_1,\ x - z = c_2$

(d) $x^3 - y^3 = c_1,\ x^2 + z^2 = c_2$

53. The necessary condition for integrability of the differential equation

$$P(x, y, z)\,dx + Q(x, y, z)\,dy + R(x, y, z)\,dz = 0$$

is

(a) $P\left(\frac{\partial Q}{\partial z} + \frac{\partial R}{\partial y}\right) + Q\left(\frac{\partial R}{\partial x} + \frac{\partial P}{\partial z}\right) + R\left(\frac{\partial P}{\partial y} + \frac{\partial Q}{\partial x}\right) = 0$

(b) $Q\left(\frac{\partial Q}{\partial z} - \frac{\partial R}{\partial y}\right) + R\left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z}\right) + P\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) = 0$

(c) $P\left(\frac{\partial Q}{\partial z} - \frac{\partial R}{\partial y}\right) + Q\left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z}\right) + R\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) = 0$

(d) $P\left(\frac{\partial Q}{\partial y} - \frac{\partial R}{\partial z}\right) + Q\left(\frac{\partial R}{\partial z} - \frac{\partial P}{\partial x}\right) + R\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) = 0$

54. For the differential equation

$$\frac{d^2y}{dx^2} + \frac{7(x+1)}{2x}\frac{dy}{dx} - \frac{3}{2x^2}y = 0$$

the point $x = 0$ is a/an

(a) regular singular point

(b) irregular singular point

(c) fixed point

(d) saddle point

55. The orthogonal trajectories of the family $y^2 = 4a(x+a)$ are the family

(a) $x^2 = 4a(y+a)$

(b) $y^2 = 4a(x+a)$

(c) $y^2 = a(x+a)$

(d) $y = 4a(x^2+a)$

56. In the third quadrant of the xy-plane, the characteristic of the differential equation

$$(1+\sqrt{xy})\,u_{xx} + 2\sqrt{1+y-xy}\;u_{xy} + (1-\sqrt{xy})\,u_{yy} + x^2u_x + y^2u_y = 0$$

is

(a) real and positive

(b) real and negative

(c) identical

(d) complex conjugates

57. The elementary solution of the Laplace equation $u_{xx} + u_{yy} = 0$ is

(a) $u(x,\ y;\ \xi, \eta) = [(x-\xi)^2 - (y-\eta)^2]/(2\pi)$

(b) $u(x,\ y;\ \xi, \eta) = [(x-\xi) + (y-\eta)]/(2\pi)$

(c) $u(x,\ y;\ \xi, \eta) = [\exp^{(x-\xi)} \sin(y-\eta)]/(2\pi)$

(d) $u(x,\ y;\ \xi, \eta) = [\log\sqrt{(x-\xi)^2 + (y-\eta)^2}]/(2\pi)$

58. The space form of a wave equation $u_{tt} = u_{xx} + u_{yy}$ under the substitution $u(x,\ y,\ t) = \psi(x,\ y)\,e^{\pm ikt}$, where k is a constant, reduces to an equation of

(a) elliptic type

(b) parabolic type

(c) hyperbolic type

(d) None of the above

59. The integral representation of the solution of the initial value problem

$$\upsilon \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \quad -\infty < x < \infty,\ t > 0$$

$$u(x, 0) = x \ , \quad -\infty < x < \infty$$

is given by

(a) $u(x, t) = \frac{1}{\sqrt{4\pi\upsilon t}} \int_{-\infty}^{\infty} e^{\frac{-(x-\xi)^2}{4\upsilon t}} d\xi$

(b) $u(x, t) = \frac{x}{\sqrt{4\pi\upsilon t}} \int_{-\infty}^{\infty} e^{\frac{-(x-\xi)^2}{4\upsilon t}} d\xi$

(c) $u(x, t) = \frac{x}{\sqrt{4\pi\upsilon t}} \int_{-\infty}^{\infty} \xi e^{\frac{-(x-\xi)^2}{4\upsilon t}} d\xi$

(d) $u(x, t) = \frac{1}{\sqrt{4\pi\upsilon t}} \int_{-\infty}^{\infty} \xi e^{\frac{-(x-\xi)^2}{4\upsilon t}} d\xi$

60. If $\bar{u}(x, s)$ represents Laplace transform of $u(x, t)$, then the initial value problem

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \quad -\infty < x < \infty,\ t > 0$$

$$u(x, 0) = 0, \quad \frac{\partial u(x, 0)}{\partial t} = \sin(\pi x)$$

transforms to

(a) $\frac{d^2\bar{u}}{dx^2} + s^2\bar{u} = \sin(\pi x)$

(b) $\frac{d^2\bar{u}}{dx^2} - s^2\bar{u} = -\sin(\pi x)$

(c) $\frac{d^2\bar{u}}{dx^2} + s\bar{u} = \sin(\pi x)$

(d) $\frac{d^2\bar{u}}{dx^2} - s\bar{u} = -\sin(\pi x)$

61. Newton's method for finding the positive square root of $R > 0$ is

(a) $\dfrac{x_i x_{i-1} + R}{x_i + x_{i-1}}$

(b) $\dfrac{1}{2}\left(x_{i+1} + \dfrac{R}{x_i}\right)$

(c) $\dfrac{1}{2}\left(x_i + \dfrac{R}{x_i}\right)$

(d) $\dfrac{2x_i^2 + x_i x_{i-1} - R}{x_i + x_{i-1}}$

62. The spectral radius of the matrix

$$\begin{bmatrix} 0 & 1/3 & 1/4 \\ -1/3 & 0 & 1/2 \\ -1/4 & -1/2 & 0 \end{bmatrix}$$

is

(a) 5/6

(b) 7/12

(c) 3/4

(d) 1/3

63. The truncation error in quadratic interpolation in an equidistant table is bounded by

(a) $\dfrac{h^2}{9\sqrt{3}} \max |f'''(\xi)|$

(b) $\dfrac{h^2}{\sqrt{3}} \max |f'''(\xi)|$

(c) $\dfrac{h^2}{9} \max |f'''(\xi)|$

(d) $\dfrac{h^2}{\sqrt{2}} \max |f'''(\xi)|$

64. A scientist uses the one-point Gauss quadrature rule based on getting exact results of integration for functions $f(x)=1$ and x. The one-point Gauss quadrature rule approximation for $\int_a^b f(x)\,dx$ is

(a) $\frac{b-a}{2}[f(a)+f(b)]$

(b) $(b-a)f\left(\frac{a+b}{2}\right)$

(c) $\frac{b-a}{2}\left[f\left(\frac{b-a}{2}\left\{-\frac{1}{\sqrt{3}}\right\}+\frac{b+a}{2}\right)+f\left(\frac{b-a}{2}\left\{\frac{1}{\sqrt{3}}\right\}+\frac{b+a}{2}\right)\right]$

(d) $(b-a)f(a)$

65. For a definite integral of any third-order polynomial, the two-point Gauss quadrature rule will give the same results as the

(a) 1-segment trapezoidal rule

(b) 2-segment trapezoidal rule

(c) 3-segment trapezoidal rule

(d) Simpson's 1/3rd rule

66. The total number of generalized coordinates of a system of two particles moving on the surface of a sphere $x^2+y^2+z^2=100$ is

(a) 1

(b) 2

(c) 3

(d) 4

67. If p and q are the generalized momentum and coordinate of a Hamiltonian system described by $H = \frac{1}{2}(p^2 - q^2)$, the equation of motion is given by

(a) $\frac{d^2q}{dt^2} + \frac{dq}{dt} + q = 0$

(b) $\frac{d^2q}{dt^2} - q = 0$

(c) $\frac{d^2q}{dt^2} - \frac{dq}{dt} - q = 0$

(d) $\frac{d^2q}{dt^2} + q = 0$

68. The canonical transformation of coordinates q and p in two-dimensional phase space is given by $\overline{q} = q - p$, $\overline{p} = q + p$. Then the corresponding Hamiltonian transformation is given by

(a) $\overline{H} = 2H$

(b) $\overline{H} = H$

(c) $\overline{H} = -2H$

(d) $\overline{H} = -H$

69. The extremal of the integral $\int_0^{\pi/2}(y'^2 - y^2)\,dx$, $y(0) = 0$ and $y\left(\frac{\pi}{2}\right) = 1$ is

(a) $\sin x + \cos x$

(b) $\sin x - \cos x$

(c) $\sin x$

(d) $\cos x$

70. The path that minimizes the arc length of the curve between $(x_0, y_0) = (0, 0)$ and $(x_1, y_1) = (1, 1)$ is

(a) $y(x) = x$

(b) $y(x) = x^2$

(c) $y(x) = x^3$

(d) $y(x) = x^4$

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