50005

QUESTION PAPER SERIES CODE
A

Test Centre : ____________________

Roll No. : ____________________

Name of the Candidate : ____________________

S A U

Entrance Test for M.Phil./Ph.D. (Applied Mathematics), 2016

[PROGRAMME CODE : PAM]

Question Paper

Time : 3 hours *Maximum Marks* : 70

INSTRUCTIONS FOR CANDIDATES

Candidates must carefully read the following instructions before attempting the Question Paper :

(i) Write your Name, Roll Number and Name of the Test Centre in the space provided for the purpose on the top of this Question Paper and on the OMR Sheet.

(ii) This Question Paper has Two Parts : Part—A and Part—B.

(iii) Part—A (Objective-type) has 30 questions of **1** mark each. All questions are compulsory. Part—B (Objective-type) has 40 questions of **1** mark each. All questions are compulsory.

(iv) **A wrong answer will lead to deduction of one-fourth of the marks assigned to that questions.**

(v) Symbols have their usual meanings.

(vi) **Please darken the appropriate circle of 'Question Paper Series Code' and 'Programme Code' on the OMR Sheet in the space provided.**

(vii) All questions should be answered on the OMR sheet.

(viii) Answers written inside the Question Paper will **NOT** be evaluated.

(ix) **Calculators and Log Tables may be used. Mobile Phones are NOT allowed.**

(x) Three pages at the end of the Question Paper have been provided for Rough Work.

(xi) **Return the Question Paper and the OMR Sheet** to the Invigilator at the end of the Entrance Test.

(xii) **DO NOT FOLD THE OMR SHEET.**

INSTRUCTIONS FOR MARKING ANSWERS ON THE 'OMR SHEET'

Use BLUE/BLACK Ballpoint Pen Only

1. **Please ensure that you have darkened the appropriate circle of 'Question Paper Series Code' and 'Programme Code' on the OMR Sheet in the space provided.**

   Example :

   Question Paper Series Code

   Write Question Paper Series Code A or B in the box and darken appropriate circle.

   ☐ A or B

   ●

   Ⓑ

   Programme Code

   Write Programme Code in the box and darken appropriate circle.

   Write Programme Code ☐

MEC	○	MAM	○	PCS	○
MSO	○	MLS	○	PBT	○
MIR	○	PEC	○	PAM	●
MCS	○	PSO	○	PLS	○
MBT	○	PIR	○		

2. Use only Blue/Black Ballpoint Pen to darken the Circle. Do not use Pencil to darken the Circle for Final Answer.
3. Please darken the whole Circle. ●
4. Darken ONLY ONE CIRCLE for each question as shown below in the example :

   Example :

Wrong	Wrong	Wrong	Wrong	Correct
● ⓑ ⓒ ●	ⓐ̸ ⓑ ⓒ ⓓ	ⓐ̸ ⓑ ⓒ ⓓ̸	⦿ ⓑ ⓒ ●	ⓐ ⓑ ⓒ ●

5. Once marked, no change in the answer is allowed.
6. Please do not make any stray marks on the OMR Sheet.
7. Please do not do any rough work on the OMR Sheet.
8. Mark your answer only in the appropriate circle against the number corresponding to the question.
9. **A wrong answer will lead to the deduction of one-fourth (¼) of the marks assigned to that question.**
10. Write your six-digit Roll Number in small boxes provided for the purpose; and also darken appropriate circle corresponding to respective digits of your Roll Number as shown in the example below.

    Example :

    ROLL NUMBER

1	3	5	7	2	0
●	①	①	①	①	①
②	②	②	②	●	②
③	●	③	③	③	③
④	④	④	④	④	④
⑤	⑤	●	⑤	⑤	⑤
⑥	⑥	⑥	⑥	⑥	⑥
⑦	⑦	⑦	●	⑦	⑦
⑧	⑧	⑧	⑧	⑧	⑧
⑨	⑨	⑨	⑨	⑨	⑨
⓪	⓪	⓪	⓪	⓪	●

PART—A

1. The sequence $< a_n >$ defined by $a_n = \sum_{j=1}^{n} \frac{(-1)^j 2}{n}$

   (a) is oscillatory

   (b) diverges to $+\infty$

   (c) diverges to $-\infty$

   (d) converges to 0

2. The value of the integral

$$\int_C \frac{(\sin^2 z + 2)(\cos z + 1)}{z(z+6)} dz$$

   where, z is complex and $C = \{z : |z| = 1\}$ equals

   (a) $\pi i / 3$

   (b) $2\pi i / 3$

   (c) $4\pi i / 3$

   (d) 0

3. If $\mathbb{R}$ denotes the set of real numbers, $\mathbb{T}$ the cofinite topology on $\mathbb{R}$ and $\mathbb{U}$ the usual topology on $\mathbb{R}$ and if $f : (\mathbb{R}, \mathbb{T}) \to (\mathbb{R}, \mathbb{U})$ and $g : (\mathbb{R}, \mathbb{U}) \to (\mathbb{R}, \mathbb{T})$ be identity maps, then which of the following is true?

   (a) f is continuous, g is discontinuous

   (b) g is continuous, f is discontinuous

   (c) f and g both are continuous

   (d) f and g both are discontinuous

4. In which of the following Banach spaces is the parallelogram law satisfied?

   (a) l_2

   (b) l_1

   (c) l_∞

   (d) $C[0, 1]$

5. If T is a continuous linear operator on a normed linear space X, then which of the following is ***not*** true?

(a) If $< x_n > \to x$ weakly in X, then $< Tx_n > \to Tx$ weakly in X

(b) If $< x_n > \to x$ strongly in X, then $< Tx_n > \to Tx$ strongly in X

(c) If $< x_n > \to x$ strongly in X, then $< Tx_n > \to Tx$ weakly in X

(d) If $< x_n > \to x$ weakly in X, then $< Tx_n > \to Tx$ strongly in X

6. If f and g be two real-valued functions defined on a closed bounded interval I and if $f = g$ almost everywhere on I, then which one of the following is ***not*** true?

(a) If f is measurable (Lebesgue), then g is also measurable on I

(b) If f is non-measurable, then g is also non-measurable on I

(c) If f is continuous, then g is also continuous on I

(d) If f is integrable, then g is also integrable on I

7. Which one of the following statements is ***not*** true?

(a) Cantor's ternary set has measure (Lebesgue) zero.

(b) The set of irrationals of the form $\sqrt{n} + \sqrt{m}$, (n and m are natural numbers) has measure zero.

(c) The set of algebraic real numbers has measure zero.

(d) The set of transcendental real numbers has measure zero.

8. The dimension of the subspace of $M_{2\times 2}$ spanned by

$$\begin{pmatrix} 1 & -5 \\ -4 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ -1 & 5 \end{pmatrix} \text{ and } \begin{pmatrix} 2 & -4 \\ -5 & 7 \end{pmatrix}$$

is

(a) 1

(b) 2

(c) 3

(d) 4

9. If A and B are n-square positive definite matrices, then which of the following is positive definite?

(a) $A+B$

(b) ABA

(c) AB

(d) A^2+I

10. If G is a group of order 91, then which of the following statements is false?

(a) G has one subgroup of order 7

(b) G has two subgroups of order 13

(c) G has subgroups of order 7 and 13

(d) None of the above

11. Let G be a group and let H and K be two subgroups of G. If both H and K have 12 elements, then which of the following numbers cannot be the cardinality of the set

$$HK = \{hk : h \in H,\ k \in K\}?$$

(a) 72

(b) 60

(c) 48

(d) 36

12. In $U(40)$, the cyclic subgroups of order 4 are

(a) 4

(b) only one

(c) at most equal to order of the group

(d) exactly two

13. If S_n be the symmetric group of n letters, then there exists an onto group homomorphism

(a) from S_5 to S_4

(b) from S_4 to S_2

(c) from S_5 to $\mathbb{Z}/5$

(d) from S_4 to $\mathbb{Z}/4$

14. For a group G, if Aut(G) denotes the group of automorphisms of G, then which of the following statements is true?

(a) Aut ($\mathbb{Z}$) is isomorphic to $\mathbb{Z}_2$

(b) If G is cyclic, then Aut (G) is cyclic

(c) If Aut (G) is trivial, then G is trivial

(d) Aut ($\mathbb{Z}$) is isomorphic to $\mathbb{Z}$

15. The general solution of the differential equation

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 8y = 10e^x \cos x$$

is

(a) $e^{-2x}(k_1 \cos 2x + k_2 \sin 2x) - e^x(2\cos x - \sin x)$

(b) $e^{-2x}(k_1 \cos 2x + k_2 \sin 2x) + e^x(2\cos x + \sin x)$

(c) $e^{2x}(k_1 \cos 2x + k_2 \sin 2x) + e^x(2\cos x - \sin x)$

(d) $e^{2x}(k_1 \cos 2x + k_2 \sin 2x) + e^x(2\cos x + \sin x)$

16. The region in which the equation $xu_{xx} + u_{yy} = x^2$ is hyperbolic is

(a) the half plane $x < 0$

(b) the whole plane $\mathbb{R}^2$

(c) the half plane $y > 0$

(d) the half plane $x > 0$

17. The general solution of the differential equation $(6x^2 - e^{-y^2})dx + 2xye^{-y^2}dy = 0$ is

(a) $x(2x^2 + e^{-y^2}) = c$

(b) $x(2x^2 - e^{-y^2}) = c$

(c) $x^2(2x + e^{-y^2}) = c$

(d) $x^2(2x - e^{-y^2}) = c$

18. The solution of the initial value problem

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 25y = 0,\ y(0) = -3,\ y'(0) = -1$$

is

(a) $y = e^{3x}(2\sin 4x - 3\cos 4x)$

(b) $y = e^{3x}(2\cos 2x + 3\sin 2x)$

(c) $y = e^{-3x}(2\sin 2x - 3\cos 2x)$

(d) $y = e^{-3x}(2\sin 4x + 3\cos 4x)$

19. The general integral of the partial differential equation $z(xp - yq) = y^2 - x^2$ is

(a) $z^2 = x^2 + y^2 + f(xy)$

(b) $z^2 = x^2 - y^2 + f(xy)$

(c) $z^2 = y^2 - x^2 + f(xy)$

(d) $z^2 = -x^2 - y^2 + f(xy)$

20. The general solution of

$$\frac{d^3y}{dx^3}+3\frac{d^2y}{dx^2}+3\frac{dy}{dx}+y=0$$

is

(a) $c_1+c_2x+c_3x^2e^{-x}$

(b) $c_1+c_2x+c_3x^2e^{x}$

(c) $(c_1+c_2x+c_3x^2)e^{-x}$

(d) $(c_1+c_2x+c_3x^2)e^{x}$

21. For the equation

$$x^2(x-2)\frac{d^2y}{dx^2}+(x+1)\frac{dy}{dx}-3xy=0$$

consider the following statements :

P : $x=0$ is a regular singular point

Q : $x=2$ is a regular singular point

Then

(a) P is false but Q is true

(b) P is true but Q is false

(c) both P and Q are true

(d) both P and Q are false

22. If Δ and ∇ are the forward and the backward difference operators respectively, then $\nabla-\Delta$ is equal to

(a) $\frac{\Delta}{\nabla}$

(b) $\Delta\nabla$

(c) $-\Delta\nabla$

(d) $\Delta+\nabla$

23. The maximum step size h is such that the error in linear interpolation for the function $y = \sin(x)$ in $[0, \pi]$ is less than 5×10^{-5} is

(a) 0·02

(b) 0·002

(c) 0·04

(d) 0·06

24. If $|\text{error}| < 10^{-4}$, then the number of iterations for finding root of $\cos(x) - e^{-x} = 0$ by bisection method in interval $\left[-\frac{1}{2}, \frac{1}{2}\right]$ is

(a) at least 10

(b) at least 14

(c) at least 17

(d) None of the above

25. A problem in statistics is given to three students whose chances of solving it are 1/2, 1/3 and 1/4 respectively. The probability that the problem will be solved is

(a) 1/8

(b) 1/24

(c) 3/4

(d) 3/8

26. The density function of a random variable x is given by $f(x) = kx(2 - x)$, $0 \le x \le 2$. Its mean is given by

(a) 1

(b) 3/4

(c) 1/2

(d) 1/4

27. The value of the moment generating function $M_0(t)$ of the exponential distribution $f(x) = 8e^{-8x}$, $0 \le x < \infty$ at $t = 3$ is given by

(a) 5/8

(b) 1/2

(c) 8/5

(d) 2

28. The solution set which satisfies the non-negativity constraint is classified as

(a) basic feasible solution

(b) feasible value solution

(c) basic solution

(d) positive-negative value

29. The primal problem has unbounded solution if

(a) dual has bounded solution

(b) dual has unbounded solution

(c) dual has no feasible solution

(d) dual has feasible solution

30. In a maximization LPP, the variable corresponding to the minimum ratio with solution column leaves the basis. This ensures

(a) the largest rise in the objective function

(b) that the next solution will be a BFS

(c) that the next solution will not be unbounded

(d) None of the above

31. Let f be defined as $f(x) = \text{Max}\{[x^2], x\}$, $x \in [0, 2]$; where $[y]$ denotes the greatest integer less than or equal to y. Then f satisfies which of the following?

(a) f is continuous at all points in $[0, 2]$ except one point

(b) f is continuous at all points in $[0, 2]$ except two points

(c) f is continuous at all points in $[0, 2]$ except three points

(d) f is continuous at all points in $[0, 2]$

32. Let f be defined as $f(x) = x^3 - 3x + 1$, $x \in [0, 1]$. Then f satisfies which of the following?

(a) $f(x) \neq 0$ for any x in $[0, 1]$

(b) $f(x)$ has exactly one zero in $[0, 1]$

(c) $f(x)$ has exactly two zeros in $[0, 1]$

(d) $f(x)$ has all the three zeros in $[0, 1]$

33. The series

$$1 + \frac{x}{1!} + \frac{2^2 x^2}{2!} + \frac{3^3 x^3}{3!} + \cdots, \; x > 0$$

is

(a) convergent for all $x > 0$

(b) divergent for $x = 1/e^3$

(c) divergent for $x = 1/e^2$

(d) divergent for $x = 1/e$

34. The sequence of functions $< f_n >$ defined on $[0, 1]$ as $f_n(x) = \frac{nx}{1 + n^3 x^2}$, $n = 1, 2, 3, \cdots$ satisfies which of the following?

(a) uniformly convergent over $[0, 1]$

(b) only pointwise but not uniformly convergent over $[0, 1]$

(c) uniformly convergent in $[0, \frac{1}{2}]$ and pointwise only for $\frac{1}{2} < x \leq 1$

(d) pointwise convergent only at $x = 0$

35. The analytic function whose imaginary part is $\frac{x-y}{x^2+y^2}$ is

(a) $\frac{iz}{1+z^2}+c$

(b) $\frac{(1+z)i}{1+z^2}+c$

(c) $\frac{1+i}{z}+c$

(d) $\frac{(e^z-1)i}{1+z}+c$

36. Residue of the function $f(z)=z\cos\left(\frac{1}{z}\right)$ *at* $z=0$ is

(a) $-1/2$

(b) $-1/3$

(c) $-1/4$

(d) 0

37. Which of the following is not true?

(a) The set of rational numbers with the usual relative topology of $\mathbb{R}$ is disconnected

(b) The set of irrational numbers with the usual relative topology of $\mathbb{R}$ is disconnected

(c) The set of real numbers $\mathbb{R}$ with the usual topology is disconnected

(d) The set of real numbers $\mathbb{R}$ with the topology generated by semi-open interval $(a, b]$ is disconnected

38. Which of the following normed linear spaces is not Banach?

(a) The space of all continuous functions on [0, 1] with norm $\|f\|=\int_0^1 |f(x)|\,dx$

(b) The space of all continuous functions on [0, 1] with norm $\|f\|=\sup\{|f(x)|: x\in[0,1]\}$

(c) The space $\mathbf{C}^n=\{(z_1, z_2, \cdots, z_n): z_i\in\mathbf{C}\}$ with norm $\|(z_1, z_2, \cdots, z_n)\|=\sqrt{\sum_{i=1}^{n}|z_i|^2}$

(d) The space of all Lebesgue integral functions on [0, 1] with norm $\|f\|=\int_0^1|f(t)|\,dt$

39. Let T be a bounded linear operator defined on $\mathbb{C}^2$ as $T(1, 0) = (0, 1)$ and $T(0, 1) = (1, 0)$. Then $\sigma(T)$, the spectrum of T equals

(a) $\{0, 1\}$

(b) $\{0, -1\}$

(c) $\{1, -1\}$

(d) $\{1\}$

40. Let $< f_n >$ be a sequence of functions defined on $[0, 1]$ as

$$f_n(x) = \begin{cases} 0\ , & 0 \le x < \dfrac{1}{2n} \\ 2n\ , & \dfrac{1}{2n} \le x \le \dfrac{1}{n} \\ 0\ , & \dfrac{1}{n} < x \le 1 \end{cases}$$

Then which of the following does not hold?

(a) $f_n \to f$ almost everywhere on $[0, 1]$

(b) Fatou's lemma holds

(c) dominated convergence theorem cannot be applied

(d) $\int_0^1 \lim_{n\to\infty} f_n(x)\,dx = \lim_{n\to\infty} \int_0^1 f_n(x)\,dx$

41. Let A and B be $n \times n$ real matrices such that $AB = BA = O$ and $A + B$ is invertible. Then which of the following may not be true?

(a) rank (A) = rank (B)

(b) rank (A) + rank (B) = n

(c) nullity (A) + nullity (B) = n

(d) $A - B$ is invertible

42. Let $T : R^3 \to R^3$ such that $T(a, b, c) = (0, a, b)$, for $(a, b, c) \in R^3$. Then $T + I$ is a zero of the polynomial

(a) t

(b) t^2

(c) t^3

(d) None of the above

43. If A and B are square matrices such that $AB = I$, then zero is an eigenvalue of

(a) A but not of B

(b) B but not of A

(c) both A and B

(d) neither A nor B

44. Let $A = (a_{ij})$ be $n \times n$ complex matrix and A^* denote the conjugate transpose of A. Then which of the following statements is false?

(a) If A is invertible, then $\operatorname{tr}(A^* A) \neq 0$, i.e., trace of $A^* A$ is nonzero

(b) If $\operatorname{tr}(A^* A) \neq 0$, then A is invertible

(c) If $|\operatorname{tr}(A^* A)| < n^2$, then $|a_{ij}| < 1$ for some i, j

(d) If $\operatorname{tr}(A^* A) = 0$, then A is zero matrix

45. If $\sigma : \{1, 2, 3, 4, 5\} \to \{1, 2, 3, 4, 5\}$ be a permutation (one-to-one and onto function) such that $\sigma^{-1}(j) \leq \sigma(j)$ for all j, $1 \leq j \leq 5$, then which of the following is false?

(a) $\sigma \circ \sigma(j) = j$ for all j, $1 \leq j \leq 5$

(b) $\sigma^{-1}(j) = \sigma(j)$ for all j, $1 \leq j \leq 5$

(c) The set $\{k : \sigma(k) \neq k\}$ has an even number of elements

(d) The set $\{k : \sigma(k) = k\}$ has an odd number of elements

46. Let G be a group. Suppose $|G| = p^2 q$, where p and q are distinct prime numbers satisfying

$$q \not\equiv 1 \bmod p$$

Then which of the following is true?

(a) G has more than one p-Sylow subgroup

(b) G has a normal p-Sylow subgroup

(c) The number of q-Sylow subgroups of G is divisible by pd

(d) G has a unique q-Sylow subgroup

47. If $C([0, 1])$ be the ring of all real-valued continuous functions on $[0, 1]$, which of the following statements is true?

(a) $C([0, 1])$ is an integral domain

(b) The set of all functions vanishing at 0 is a maximal ideal

(c) The set of all functions vanishing at both 0 and 1 is a prime ideal

(d) If $f \in C([0, 1])$ is such that $(f(x))^n = 0$ for all $x \in [0, 1]$ for some $n > 1$, then $f(x) = 0$ for all $x \in [0, 1]$

48. Which of the following polynomials is irreducible over the indicated rings?

(a) $x^5 - 3x^4 + 2x^3 - 5x + 8$ over $\mathbb{R}$

(b) $x^4 + x^2 + 1$ over $\mathbb{Z}/2\mathbb{Z}$

(c) $x^3 + 3x^2 - 6x + 3$ over $\mathbb{Z}$

(d) None of the above

49. Let PID, ED, UFD denote the set of all principal ideal domains, Euclidean domains, unique factorization domains, respectively. Then

(a) $\text{UFD} \subset \text{ED} \subset \text{PID}$

(b) $\text{PID} \subset \text{ED} \subset \text{UFD}$

(c) $\text{ED} \subset \text{PID} \subset \text{UFD}$

(d) $\text{PID} \subset \text{UFD} \subset \text{ED}$

50. The solution of the Cauchy problem for the first-order partial differential equation

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = z, \text{ on } D = \{(x, y, z) : x^2 + y^2 \neq 0, z > 0\}$$

with the initial conditions $x^2 + y^2 = 1$, $z = 1$ is

(a) $z = x^2 + y^2$

(b) $z = (x^2 + y^2)^2$

(c) $z = (x^2 + y^2)^{\frac{1}{2}}$

(d) $z = (2 - (x^2 + y^2))^{\frac{1}{2}}$

51. The orthogonal trajectories of the system of parabolas $y^2 = 4a(x+a)$ belong to

(a) the system of circles $x^2 + y^2 = a^2$

(b) the system of hyperbolas $xy = a^2$

(c) the system of parabolas $y^2 = 4a(x+a)$

(d) None of the above

52. Applying Charpit's method, the solution of the equation $px + qy = pq$ is

(a) $az = \frac{1}{2}(ax + y) + b$

(b) $az = \frac{1}{2}(ay + x) + b$

(c) $az = \frac{1}{2}(ay + x)^{\frac{1}{2}} + b$

(d) $az = \frac{1}{2}(ax + y)^2 + b$

53. The general solution of the second-order partial differential equation

$$u_{xx} + u_{xy} - 2u_{yy} = (y+1)e^x$$

is given by

(a) $\phi_1(y+x) + \phi_2(y-2x) + ye^x$

(b) $\phi_1(y+x) + \phi_2(y-2x) + xe^{-y}$

(c) $\phi_1(y-x) + \phi_2(y+2x) + ye^{-x}$

(d) $\phi_1(y-x) + \phi_2(y+2x) + xe^y$

54. The solution of the total differential equation $(x^2z - y^3)dx + 3xy^2dy + x^3dz = 0$ is given by

(a) $xy + \frac{z^3}{x} = c$

(b) $xz + \frac{y^3}{x} = c$

(c) $yz + \frac{z^3}{y} = c$

(d) $xz + \frac{x^3}{y} = c$

55. The partial differential equation $\frac{\partial^2 u}{\partial y^2} - y\frac{\partial^2 u}{\partial x^2} = 0$ has

(a) no real characteristics for $y > 0$

(b) two families of real characteristic curves for $y < 0$

(c) branches of quadratic curves as characteristics for $y \neq 0$

(d) vertical lines as a family of characteristic curves for $y = 0$

56. The general integral of the partial differential equation

$$(x^2 - y^2 - z^2)\frac{\partial z}{\partial x} + 2xy\frac{\partial z}{\partial y} = 2xz$$

is

(a) $f\left(\frac{y}{z}, \frac{x^2 + y^2 + z^2}{x}\right) = 0$

(b) $f\left(\frac{y}{z}, \frac{x^2 + y^2 + z^2}{z}\right) = 0$

(c) $f\left(\frac{xy}{z}, \frac{y^2}{x^2 + z^2}\right) = 0$

(d) $f\left(\frac{y}{zx}; \frac{x^2}{y^2 + z^2}\right) = 0$

57. If $J_n(x)$ defines the Bessel's function of the first kind and of order n, then which of the following is true?

(a) $\frac{d}{dx}(x^{-n}J_n(x)) = x^{-n}$

(b) $\frac{d}{dx}(x^{-n}J_n(x)) = J_{n+1}(x)$

(c) $\frac{d}{dx}(x^{-n}J_n(x)) = -x^{-n}J_{n+1}(x)$

(d) $\frac{d}{dx}(x^{-n}J_n(x)) = 0$

58. The partial differential equation for the family of surfaces $z = ce^{\omega t}\sin(\omega x)$, where c and ω are arbitrary constants is

(a) $z_{xx} - z_{tt} = 0$

(b) $z_{xx} + z_{tt} = 0$

(c) $z_{xt} + z_{tt} = 0$

(d) $z_{xt} + z_{xx} = 0$

59. The interpolating polynomial for the function satisfying the data

$$f(-1) = -2,\ f(0) = -1,\ f(2) = 7,\ f'(-1) = -3,\ f'(0) = 4,\ f'(2) = 36$$

is

(a) $p(x) = x^5 + 4x^3 + 4x - 1$

(b) $p(x) = x^5 - 4x^3 + 4x - 1$

(c) $p(x) = x^5 - 4x^3 - 4x - 1$

(d) None of the above

60. The second-order Runge-Kutta method with step-size 0·5 applied to the equation

$$\frac{dy}{dx} = 1 + \frac{y}{x},\ y(1) = 1$$

gives the approximate solution

(a) $y(2\cdot0) = 1\cdot39$

(b) $y(2\cdot0) = 2\cdot39$

(c) $y(2\cdot0) = 3\cdot39$

(d) $y(2\cdot0) = 4\cdot39$

61. The coefficients in the three-step multistep method

$$y_{i+1} - y_i = h[a_0 f(x_i, y_i) + a_1 f(x_{i-1}, y_{i-1})]$$

for the equation $y'(x) = f(x, y)$ are

(a) $a_0 = 2, a_1 = 3$

(b) $a_0 = 3/2, a_1 = -1/2$

(c) $a_0 = 2/3, a_1 = 3/2$

(d) $a_0 = 1, a_1 = 1$

62. The least square polynomial approximation of degree one for $f(x) = x^3$ on the interval $[0, 1]$ with weight function 1 is

(a) $\frac{9x-2}{5}$

(b) $\frac{2-9x}{5}$

(c) $\frac{2-9x}{10}$

(d) $\frac{9x-2}{10}$

63. A man parks his car among 27 cars in a row not at either end. On his return, he finds that 12 places are still occupied. The probability, that both neighbouring places are empty, is

(a) 12/25

(b) 21/65

(c) 6/13

(d) 13/25

64. A random variable x can assume any positive integral value n with a probability proportional to $\frac{1}{3^n}$. Then $E(x)$ equals

(a) 3/2

(b) 4/3

(c) 5/4

(d) 1

65. A and B alternately throw a pair of dice. The one who throws 9 first wins. The chances of their winning are in the ratio

(a) 6 : 5

(b) 7 : 6

(c) 8 : 7

(d) 9 : 8

66. The frequency distribution of a measurable characteristic varying between 0 and 2 is

$$f(x) = \begin{cases} x^3 & , \quad 0 \le x \le 1 \\ (2-x)^3 & , \quad 1 \le x \le 2 \end{cases}$$

The mean deviation about the mean is

(a) 1/3

(b) 1/4

(c) 1/5

(d) 1/6

67. Addition of a new constraint to an LPP can affect

(a) feasibility condition only

(b) optimality condition only

(c) feasibility and optimality conditions both

(d) neither feasibility nor optimality conditions

68. If the primal constraint is originally in equation form, then the corresponding dual variable is necessarily

(a) non-negative

(b) positive

(c) unrestricted

(d) None of the above

69. Pick the wrong statement.

(a) The dual of the dual is primal.

(b) An equation in a constraint of a primal problem implies the associated variable in the dual problem to be unrestricted in sign.

(c) If a primal variable is non-negative, the corresponding dual constraint is an equation.

(d) The objective function coefficients in the primal problem become right-hand side of constraints of the dual.

70. Change in availability vector and addition of new constraint, simultaneously to an LPP

(a) may disturb feasibility

(b) may disturb optimality

(c) may disturb both feasibility and optimality

(d) None of the above

SPACE FOR ROUGH WORK