Test Centre : ______________________

Roll No. : ______________________

Name of the Candidate : ______________________

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Entrance Test for Ph.D. (Applied Mathematics) 2018

[PROGRAMME CODE : 50005]

Question Paper Series Code : A

QUESTION PAPER

Time : 3 hours *Maximum Marks* : 70

INSTRUCTIONS FOR CANDIDATES

Candidates must carefully read the following instructions before attempting the Question Paper :

(i) Write your Name, Roll Number and Name of the Test Centre in the space provided for the purpose on the top of this Question Paper and on the OMR Sheet.

(ii) This Question Paper has Two Parts : Part—A and Part—B.

(iii) Part—A (objective-type) has 30 questions of 1 mark each. All questions are compulsory. Part—B (objective-type) has 40 questions of 1 mark each. All questions are compulsory.

(iv) **A wrong answer will lead to the deduction of one-fourth (¼) of the marks assigned to that question.**

(v) Symbols have their usual meanings.

(vi) **Please darken the appropriate circle of 'Question Paper Series Code' and 'Programme Code' on the OMR Sheet in the space provided.**

(vii) All questions should be answered on the OMR sheet.

(viii) Answers written inside the Question Paper will **NOT** be evaluated.

(ix) **Non-programmable calculators and Log Tables may be used. Mobile Phones are NOT allowed.**

(x) Pages at the end of the Question Paper have been provided for rough work.

(xi) **Return the Question Paper and the OMR Sheet** to the Invigilator at the end of the Entrance Test.

(xii) **DO NOT FOLD THE OMR SHEET.**

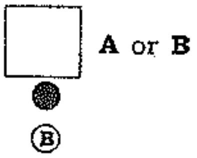
INSTRUCTIONS FOR MARKING ANSWERS ON THE 'OMR SHEET'

Use BLUE/BLACK Ballpoint Pen Only

1. **Please ensure that you have darkened the appropriate circle of 'Question Paper Series Code' and 'Programme Code' on the OMR Sheet in the space provided.**

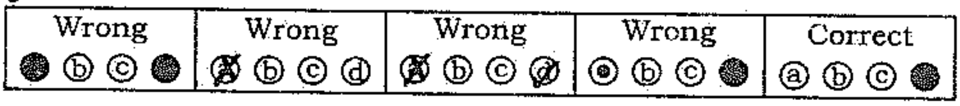
   Question Paper Series Code

   Write Question Paper Series Code **A** or **B** in the box and darken the appropriate circle.



2. Use only Blue/Black Ballpoint Pen to darken the circle. Do not use Pencil to darken the circle for Final Answer.
3. Please darken the whole circle.
4. Darken ONLY ONE CIRCLE for each question as shown below in the example :

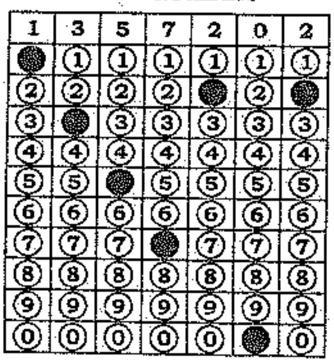
   Example :



5. Once marked, no change in the answer is possible.
6. Please do not make any stray marks on the OMR Sheet.
7. Please do not do any rough work on the OMR Sheet.
8. Mark your answer only in the appropriate circle against the number corresponding to the question.
9. **A wrong answer will lead to the deduction of one-fourth of the marks assigned to that question.**
10. Write your seven-digit Roll Number in small boxes provided for the purpose; and also darken the appropriate circle corresponding to respective digits of your Roll Number as shown in the example below.

    Example :

    ROLL NUMBER



PART—A

1. The sequence $\{f_n\}$ of functions is defined by $f_n(x) = 2^n x^n$. In order that the sequence $\{f_n\}$ is uniformly convergent in the interval $[0, k]$

   a. $k < 1$

   b. $k < 1/2$

   c. $k < 3/2$

   d. $k < 2$

2. If $\langle x_n \rangle_{n=1}^{\infty}$ is a sequence defined as $x_n = \sum_{m=1}^{n} \frac{1}{m^2}$, then the sequence $\langle x_n \rangle$ ***does not*** satisfy which one of the following statements?

   a. $\langle x_n \rangle$ is a Cauchy sequence

   b. $\langle x_n \rangle$ is a unbounded sequence

   c. $\langle x_n \rangle$ is a monotonically increasing sequence

   d. $\langle x_n \rangle$ is a convergent sequence

3. At the point (0, 0), the function

$$f(x, y) = \begin{cases} \frac{xy}{\sqrt{x^2 + y^2}}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

   a. is differentiable

   b. does not have partial derivatives

   c. is continuous but not differentiable

   d. is not continuous

4. The series $\sum_{n=1}^{\infty} \frac{(n+1)^n x^n}{n^{n+1}}$, $x > 0$, ***does not*** satisfy which one of the following?

   a. Divergent if $x > 1$

   b. Convergent if $x < 1$

   c. Divergent if $x = 1$

   d. Convergent if $x = 1$

5. Which one of the following sequences $\langle f_n(x) \rangle$, $n = 1, 2, 3, \cdots$, of functions is uniformly convergent over $[0, 1]$?

a. $f_n(x) = \dfrac{nx}{1+n^2x^2}$

b. $f_n(x) = \dfrac{nx}{1+n^3x^3}$

c. $f_n(x) = \dfrac{nx}{1+n^3x^2}$

d. $f_n(x) = \dfrac{nx}{1+n^2x^3}$

6. If $f(z) = \sqrt{|xy|}$, for all $z = x + iy$, a complex number, then which one of the following is ***not*** satisfied?

a. f is continuous at $z = 0$

b. Cauchy-Riemann equations are satisfied at $z = 0$

c. $|f(z)| \leq 1$ for all z satisfying $|z| \leq 1$

d. f is analytic at $z = 0$

7. Which one of the following is a metric on R?

a. $d(x, y) = \min\{1, |x - y|\}$

b. $d(x, y) = |x^2 - y^2|$

c. $d(x, y) = \sin|x - y|$

d. $d(x, y) = |x - y|^2$

8. The system of equations

$$5x + 7y = b_1$$
$$2x + 3y = b_2$$

is consistent for

a. at least one b_1 and b_2

b. all b_1 and b_2

c. no b_1 and b_2

d. exactly one b_1 and b_2

9. Which one of the following is ***not*** true for a square matrix A?

a. The eigenvalues of A and its transpose are the same

b. The sum of the eigenvalues of A is the sum of the elements on its principal diagonal

c. The product of the eigenvalues of A is the product of the elements on its principal diagonal

d. If A is idempotent then its eigenvalues are either 0 or 1

10. If G is a cyclic group of order 20, then the number of generators of G is

a. 7

b. 8

c. 9

d. 10

11. There exists a field having

a. 8 elements

b. 7 elements

c. 6 elements

d. 5 elements

12. In the ring $\mathbb{Z}[\sqrt{5}] = \{a + b\sqrt{5} \mid a, b \in \mathbb{Z}\}$, which one of the following is correct?

a. 2 and $1+\sqrt{5}$ are irreducible and prime

b. 2 and $1+\sqrt{5}$ are irreducible but not prime

c. 2 and $1+\sqrt{5}$ are not irreducible but prime

d. 2 and $1+\sqrt{5}$ are neither irreducible nor prime

13. If V be the vector space of 5×5 matrices over the field of real numbers, then which one of the following sets is ***not*** a subspace of V?

a. All lower triangular matrices of order 5

b. All nilpotent matrices of order 5

c. All diagonal matrices of order 5

d. All scalar matrices of order 5

14. If the subspace W is defined as $W = \{(a, b, 0) \mid a, b \in R\}$, then

a. $W^{\perp} = \{(x, 0, 0) \mid x \in R\}$

b. $W^{\perp} = \{(0, y, 0) \mid y \in R\}$

c. $W^{\perp} = \{(0, 0, z) \mid z \in R\}$

d. None of the above

15. The solution of the initial value problem

$$\frac{dy}{dx} = 1 + y^2, \; y(0) = 1$$

will be given by

a. $y = \tan\left(x - \frac{\pi}{2}\right)$

b. $y = \tan\left(x + \frac{\pi}{2}\right)$

c. $y = \tan\left(x - \frac{\pi}{4}\right)$

d. $y = \tan\left(x + \frac{\pi}{4}\right)$

16. One particular solution of

$$\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} - \frac{dy}{dx} + y = -e^x$$

is a constant multiple of

a. xe^{-x}

b. xe^{x}

c. x^2e^{-x}

d. x^2e^{x}

17. The general solution of the differential equation

$$(2x + y + 1)\,dx + (4x + 2y - 1)\,dy = 0$$

is

a. $\ln(2x + y - 1) + x + 2y = k$

b. $\ln(2x + y - 1) - x - 2y = k$

c. $\ln(2x + y - 1) - x + y = k$

d. $\ln(2x + y + 1) + x - 2y = k$

18. The solution of the differential equation

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = x^3$$

is

a. $c_1 x + \frac{c_2}{x} + \frac{x}{2}$

b. $c_1 x + \frac{c_2}{x} + \frac{x^2}{4}$

c. $c_1 x + \frac{c_2}{x} + \frac{x^3}{8}$

d. $c_1 x + \frac{c_2}{x} + \frac{x^3}{16}$

19. If $u(x, t)$ is the solution of the wave equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2},\ u(x, 0) = \cos(5\pi x),\ \frac{\partial u}{\partial t}(x, 0) = 0$$

then the value of $u(1, 1)$ is

a. -1

b. 0

c. 1

d. 2

20. If P_n denotes Legendre polynomial defined on the interval $[-1, 1]$, then the value of

$$\int_{-1}^{1} (P_n(x))^2 \, dx$$

is

a. 0

b. 1

c. $2n^2$

d. $\frac{2}{2n+1}$

21. The partial differential equation arising from the surface $z = f(x^2 + y^2)$ is

a. $yq - xp = 0$

b. $yq + xp = 0$

c. $yp - xq = 0$

d. $yp + xq = 0$

22. For $f(x) = \frac{1}{x^2}$, the first divided difference $f[a, b]$ with respect to the points a and b is equal to

a. $\frac{a-b}{a^2 b^2}$

b. $\frac{a+b}{a^2 b^2}$

c. $\frac{-(a+b)}{ab}$

d. $\frac{-(a+b)}{a^2 b^2}$

23. The method $x_{n+1} = x_n - \left(\frac{x_n - x_0}{f_n - f_0}\right) f_n,\ n = 1, 2, \cdots$ is used to find a simple root of $f(x) = 0$. The order of convergence of the method is

a. 1

b. 2

c. 3

d. 1·68

24. The value of integral $\int_0^1 \frac{x}{x^2 + 10} dx$ using Simpson's $\frac{1}{3}$rd rule with $h = 0.5$ is

a. $\frac{47}{902}$

b. $\frac{45}{902}$

c. $\frac{43}{902}$

d. $\frac{41}{902}$

25. A bag X contains 2 white and 3 red balls, and a bag Y contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and is found to be red. The probability that it was drawn from Y is

a. 22/52

b. 25/52

c. 27/52

d. 15/52

26. Two numbers a and b are chosen at random from the set of the first 30 natural numbers. The probability that $a^2 - b^2$ is divisible by 3 is

a. 9/87

b. 12/87

c. 15/87

d. 47/87

27. If X is a continuous random variable with probability density function given by

$$f(x) = \begin{cases} kx, & 0 \le x < 2 \\ 2k, & 2 \le x < 4 \\ 6k - kx, & 4 \le x < 6 \end{cases}$$

The mean value of X is

a. 3

b. 2

c. 1

d. 4

28. In a maximization linear programming problem the variable corresponding to minimum ratio with solution column leaves the basis. This ensures

a. largest rise in the objective function

b. that the next solution will be a basic feasible solution

c. that the next solution will not be unbounded

d. None of the above

29. Consider the following linear programming problem :

$$\text{Maximize } Z = x_1 + 3x_2$$

subject to

$$x_1 + x_2 \leq 2$$
$$-x_1 + x_2 \leq 4$$

x_1 is unrestricted; $x_2 \geq 0$

Then which of the following is the best basic feasible objective function value?

a. 8

b. 11

c. 6

d. 15

30. For the problem

$$\text{Maximize } Z = (2x_1 + x_2 + 5x_3 + 6x_4)$$

subject to

$$2x_1 + x_2 + x_4 \leq 8$$
$$2x_1 + 2x_2 + x_3 + 2x_4 \leq 12$$

for all $x_i > 0$

It is known that $x_1 = 0, x_2 = 0, x_3 = 4, x_4 = 4$ is the optimal solution. The optimal of the dual is (if y_1 and y_2 are dual variables)

a. $y_1 = 1,\ y_2 = 4$

b. $y_1 = 4,\ y_2 = 4$

c. $y_1 = 4,\ y_2 = 1$

d. $y_1 = 0,\ y_2 = 0$

PART—B

31. The dual of the space $L^{2/3}$ is the space

a. $L^{3/2}$

b. L^2

c. L^{-2}

d. does not exist

32. The operator $T: R^2 \to R^2$ defined by $T(x_1, x_2) = (0, x_2)$ is

a. bounded but not continuous

b. continuous but not bounded

c. bounded as well as continuous

d. neither bounded nor continuous

33. Defined $f(z) = z^3 \sin\frac{1}{z}$, $z \neq 0$ and $f(0) = 0$, where z is a complex number. Then for $f(z)$, which one of the following is true?

a. $z = 0$ is a removable singularity

b. $z = 0$ is a pole of order 2

c. $z = 0$ is a pole of order 3

d. $z = 0$ is an essential singularity

34. Let S be a unit sphere in R^3, $x^2 + y^2 + z^2 = 1$. Let P be a plane in R^3, $lx + my + nz = p$ intersecting the sphere in a circle Γ. If Γ passes through the point $N(0, 0, 1)$, then its stereographic projection on the complex plane is

a. a straight line

b. a circle

c. an arc of a circle

d. an ellipse with latus rectum twice the radius of the circle Γ

35. If C be the unit circle centered at the origin, then the value of the integral $\int_C \frac{e^z}{z^3}\,dz$ is

a. $e^{\pi i}$

b. πi

c. $-\pi i$

d. $e^{-\pi i}$

36. If (X, T) be a topological space, then which one of the following is ***not*** equivalent to others?

a. X can be written as the disjoint union of two nonempty closed subsets

b. X can be written as the disjoint union of two nonempty open sets

c. There exists a subset A of X, $A \neq \phi$, $A \neq X$; A is both open and closed

d. X is a connected space

37. Which one of the following statements is ***not*** true?

a. Composite of two measurable functions is measurable.

b. If f is measurable, then $|f|$ is also measurable.

c. The sum of two measurable functions is measurable.

d. If f and g are two measurable functions, then both $\max(f(x), 1)$ and $\min(g(x), 1)$ are measurable functions.

38. Let A denote a set of algebraic numbers. Consider the function $f : [0, 2] \to R$ defined by

$$f(x) = \begin{cases} 0, & x \in A \cap [0, 2] \\ 1, & x \in [0, 2] - A \end{cases}$$

Then f is

a. only Riemann integrable

b. only Lebesgue integrable

c. Riemann as well as Lebesgue integrable

d. neither Riemann nor Lebesgue integrable

39. Which one of the following is ***not*** true for projections on Hilbert spaces?

a. The sum of two projections P and Q is a projection if and only if $PQ = QP$

b. The product of two projections P and Q is a projection if and only if $PQ = QP$

c. P-Q is a projection for projections P and Q if and only if $Q \leq P$

d. P is a projection if and only if I-P is so

40. S_3, the symmetric group of degree 3 has

a. no Sylow 2-subgroup

b. two Sylow 2-subgroup

c. three Sylow 2-subgroup

d. four Sylow 2-subgroup

41. Which of the following is ***not*** true for a group G?

a. If $O(G) = 4$, then G is cyclic

b. If $O(G) = 5$, then G is cyclic

c. If $O(G) = 6$, then either G is Abelian or isomorphic to S_3

d. If $O(G) = 8$, then either G is Abelian or isomorphic to group of quaternions

42. If $R = \{0, 2, 4, 6\}$ be the ring of integers modulo 8, and $M = \{0, 4\}$, then which one of the following is true?

a. M is a maximal ideal of R but not prime

b. M is a prime ideal of R but not maximal

c. M is both maximal and prime ideal of R

d. M is neither maximal nor prime ideal of R

43. If Q is a field of rationals, then which of the following is ***not*** true?

a. $x^2 + 1$ is irreducible over Q

b. $x^3 - 9x + 5$ is irreducible over Q

c. $x^2 + 2x + 3$ is irreducible over Z_5

d. $x^3 - 9$ is irreducible over Z_{11}

44. If U and W are distinct four-dimensional subspaces of a vector space V with $\dim(V) = 6$, then

a. $\dim(U \cap W)$ is 3 or 2

b. $\dim(U \cap W)$ is 3 or 4

c. $\dim(U \cap W)$ is 3 or 5

d. $\dim(U \cap W)$ is 3 or 6

45. If $L : M_{22} \to R$ is a linear transformation given by $L(A) = \text{trace}(A)$, then

a. $\dim(\ker(L)) = 1$

b. $\dim(\ker(L)) = 2$

c. $\dim(\ker(L)) = 3$

d. $\dim(\ker(L)) = 4$

46. Which of the following is ***not*** true?

a. The ring of integers has characteristic 0

b. The ring of even integers has characteristic 2

c. The field of rationals has characteristic 0

d. The field $Z_2 = \{0, 1\}$ of integers modulo 2 has characteristic 2

47. If $T: R^4 \to R^4$ is a linear mapping defined by

$$T = \begin{bmatrix} 0 & 0 & 0 & 0 \\ a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \end{bmatrix}$$

then T is diagonalizable, when

a. $a = 0,\ b = c = 1$

b. $b = 0,\ a = c = 1$

c. $a = b = c = 1$

d. $a = b = c = 0$

48. The complete integral of the partial differential equation $p^2 y(1 + x^2) = qx^2$ is

a. $z = a\sqrt{1 + x^2} + \frac{1}{2}a^2 y^2 + b$

b. $z = a(1 + x^2) + \frac{1}{2}ay + b$

c. $z = a(1 + x^2) + \frac{1}{2}a^2 y^2 + b$

d. $z = \frac{1}{2}a^2\sqrt{1 + x^2} + ay + b$

49. If the initial value problem

$$\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0,\ u(0, y) = 4e^{-2y}$$

then the value of $u(1, 1)$ is

a. $4e^{-4}$

b. $4e^{-2}$

c. $4e^{2}$

d. $4e^{4}$

50. The general solution of the system

$$y_1' = 3y_2$$
$$y_2' = 12y_1$$

is

a. $y_1 = c_1e^{-3t} + c_2e^{3t},\ y_2 = -2c_1e^{-3t} + 2c_2e^{3t}$

b. $y_1 = c_1e^{-6t} + c_2e^{6t},\ y_2 = -2c_1e^{-6t} + 2c_2e^{6t}$

c. $y_1 = c_1e^{-3t} + c_2e^{6t},\ y_2 = -2c_1e^{-3t} + 2c_2e^{6t}$

d. $y_1 = c_1e^{3t} + c_2e^{6t},\ y_2 = -2c_1e^{3t} + 2c_2e^{6t}$

51. Consider the diffusion equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t},\ 0 < x < \pi,\ t > 0$$

subject to the initial and boundary conditions $u(x, 0) = 4\sin 2x,\quad 0 < x < \pi$ and $u(0, t) = u(\pi, t) = 0,\ t > 0$. Then the value of $u(\pi/8, 1)$ is

a. $\frac{4}{\sqrt{e}}$

b. $\frac{4}{e^2}$

c. $\frac{4e^{-4}}{\sqrt{2}}$

d. $\frac{4e^{-9}}{\sqrt{2}}$

52. If the partial differential equation is given by

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 0$$

then which one of the following is ***not*** correct?

a. It is a second-order parabolic equation

b. The characteristic curves are given by $y = cx$, a family of straight lines passing through origin

c. The canonical form is $\frac{\partial^2 u}{\partial \eta^2} = 0$

d. The canonical form is $\frac{\partial^2 u}{\partial \eta^2} = 1$

53. The two linearly independent solutions f_1 and f_2 of the second-order homogeneous linear differential equation $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$ which are such that $f_1(0) = 1$, $f_1'(0) = 0$, $f_2(0) = 0$ and $f_2'(0) = 1$ are

a. $f_1(x) = 2e^x - e^{2x}$, $f_2(x) = -e^x + e^{2x}$

b. $f_1(x) = 2e^{-x} - e^{2x}$, $f_2(x) = -e^{-x} + e^{2x}$

c. $f_1(x) = 2e^x - e^{-2x}$, $f_2(x) = -e^x + e^{-2x}$

d. $f_1(x) = e^x + 2e^{2x}$, $f_2(x) = e^x + e^{2x}$

54. The initial boundary value problem

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t},\ 0 < x < 1,\ t > 0,\ u(0, t) = u(1, t) = 0;\ u(x, 0) = x(1-x),\ 0 \le x \le 1$$

has solution $u(x, t) = \sum_{n=1}^{\infty} b_n e^{-n^2\pi t} \sin(n\pi x)$, where b_n is equal to

a. $\frac{4}{n^3\pi^3}$, if n is odd and 0 if n is even

b. $\frac{4}{n^3\pi^3}$, if n is even and 0 if n is odd

c. $\frac{8}{n^3\pi^3}$, if n is odd and 0 if n is even

d. $\frac{8}{n^3\pi^3}$, if n is even and 0 if n is odd

55. The initial value problem $\frac{dy}{dx} = y^{1/3}$, $y(0) = 0$ has

a. no solution

b. infinitely many solutions

c. more than one but only finitely many solutions

d. a unique solution

56. If

$$\begin{pmatrix} 1 & 4 & 3 \\ 2 & 7 & 9 \\ 5 & 8 & p \end{pmatrix} = \begin{pmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & -53 \end{pmatrix} \begin{pmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{pmatrix}$$

then the value of p is

a. -2

b. -1

c. 1

d. 2

57. If Δ and ∇ are the forward and the backward difference operators, respectively, then $\Delta + \nabla$ is equal to

a. $-\Delta\nabla$

b. Δ / ∇

c. $\Delta / \nabla - \nabla / \Delta$

d. $\Delta / \nabla + \nabla / \Delta$

58. A Runge-Kutta method for solving the initial value problem $\frac{dy}{dx} = f(x, y)$, $y(x_0) = f(x_0)$ is given by $y(x+h) = y(x) + a_1 hf(x, y) + a_2 hf(x + p_1 h, y + p_2 hf(x, y))$. The values of a_1, a_2, p_1 and p_2 for this formula to be second-order accurate are

a. $a_1 = \frac{1}{3}, a_2 = \frac{2}{3}, p_1 = \frac{3}{2}, p_2 = \frac{3}{4}$

b. $a_1 = \frac{3}{4}, a_2 = \frac{1}{4}, p_1 = \frac{1}{2}, p_2 = \frac{1}{2}$

c. $a_1 = 2, a_2 = 1, p_1 = \frac{1}{2}, p_2 = \frac{1}{2}$

d. $a_1 = \frac{1}{2}, a_2 = \frac{1}{2}, p_1 = 1, p_2 = 1$

59. The iterative formula to obtain the approximate value of $\alpha^{1/3}$ of a given positive real number α using Newton-Raphson method is

a. $x_{n+1} = \frac{2}{3} x_n + \frac{\alpha}{3x_n^2}$

b. $x_{n+1} = \frac{2}{3} x_n - \frac{\alpha}{3x_n^2}$

c. $x_{n+1} = \frac{3}{2} x_n + \frac{\alpha}{3x_n^2}$

d. $x_{n+1} = \frac{3}{2} x_n - \frac{\alpha}{3x_n^2}$

60. If $f(x) = \alpha_0 + \alpha_1 x + \alpha_2 x^2$ is a polynomial, where $\alpha_0, \alpha_1, \alpha_2$ are real numbers with $\alpha_2 \neq 0$ and

$$E_1 = \int_0^1 f(x)\,dx - f\left(\frac{1}{2}\right), \quad E_2 = \int_0^1 f(x)\,dx - \frac{1}{2}(f(0) + f(1))$$

then

a. $|E_1| = |E_2|$

b. $|E_1| = 2|E_2|$

c. $|E_1| > |E_2|$

d. $|E_2| > |E_1|$

61. The probability of obtaining 3 defectives in a sample of size 10 taken without replacement from a box of 20 components containing 4 defectives is

a. 25/100

b. 20/100

c. 30/100

d. 15/100

62. If X and Y are standardized random variables where $r(2X + 3Y, 3X + 2Y) = 1$, then the coefficient of correlation $r(X, Y)$ is

a. 1/3

b. 1

c. 2/3

d. 1/2

63. If X is a continuous random variable with probability density function defined by

$$f(x) = \begin{cases} ax & , \ 0 \le x \le 1 \\ a & , \ 1 \le x \le 2 \\ -ax + 3a & , \ 2 \le x \le 3 \\ 0 & , \ \text{elsewhere} \end{cases}$$

then $P(x \le 1{\cdot}5)$ is given by

a. 1/4

b. 1/2

c. 2/3

d. 3/4

64. A coin is tossed until a head appears. The expectation of the number of tosses required will be

a. 2

b. 8

c. 16

d. 64

65. The regression equation of Y on X for the following distribution

$$f(x, y) = \frac{y e^{\frac{-y}{1+x}}}{(1+x)^4}, \quad x, y > 0$$

is

a. $y = 3(1+x)$

b. $y = 2(1+x)$

c. $y = (1+3x)$

d. $y = 3(1+2x)$

66. If in a simplex table the relative cost $(z_j - c_j)$ is zero for a non-basic variable, then there exists an alternative optimal solution, provided

a. it is a starting simplex table

b. it is an optimal simplex table

c. it can be any simplex table

d. zero relative cost $(z_j - c_j)$ for a non-basic variable, does not imply an alternative optimal solution

67. If in Phase-1 of the simplex method, an artificial variable remains at positive level in the optimal table of Phase-1, then

a. the solution is unbounded

b. there exists an optimal solution

c. there exists no solution

d. the solution is bounded

68. In canonical form of a linear programming problem, the availability of vector ***b***

a. is restricted to > 0

b. is restricted to < 0

c. is equal to 0

d. has no restriction on > 0, < 0 or = 0

69. For a linear programming problem, which of the following is a correct relation between number of basic feasible solutions (BFS) and number of vertices?

a. Number of BFS $\leq$ Number of vertices

b. Number of BFS $\geq$ Number of vertices

c. Number of BFS = Number of vertices

d. None of the above

70. The intersection of all convex sets of which a set S is a subset, is known as

a. convex hull

b. hyperplane

c. supporting hyperplane

d. None of the above

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