Sample Question Paper for PhD Applied Mathematics

Details of Syllabus

Format of the Entrance Test Paper

- The duration of the Entrance Test will be 2 hours and the question paper will consist of 50 multiple choice questions.

  - **Analysis:** Real functions; limit, continuity, differentiability; sequences; series; uniform convergence; functions of complex variables; analytic functions, complex integration; singularities, power and Laurent series; metric spaces; stereographic projection; topology, compactness, connectedness; normed linear spaces, inner product spaces; dual spaces, linear operators; Lebesgue measure and integration; convergence theorems.

  - **Algebra:** Basic theory of matrices and determinants; eigen values and eigen vectors; Groups and their elementary properties; subgroups, normal subgroups, cyclic groups, permutation groups; Lagrange's theorem; quotient groups, homomorphism of groups; Cauchy Theorem and p-groups; the structure of groups; Sylow's theorems and their applications; rings, integral domains and fields; ring homomorphism and ideals; polynomial rings and irreducibility criteria; vector space, vector subspace, linear independence of vectors, basis and dimensions of a vector space, inner product spaces, orthonormal basis; Gram-Schmidt process, linear transformations.

  - **Differential Equations:** First order ordinary differential equations (ODEs); solution of first order initial value problems; singular solution of first order ODEs; system of linear first order ODEs; method of solution of dx/P=dy/Q=dz/R; orthogonal trajectory; solution of Pfaffian differential equations in three variables; linear second order ODEs; Sturm-Liouville problems; Laplace transformation of ODEs; series solutions; Cauchy problem for first order partial differential equations (PDEs); method of characteristics; second order linear PDEs in two variables and their classification; separation of variables; solution of Laplace, wave and diffusion equations; Fourier transform and Laplace transform of PDEs.

  - **Numerical Analysis:** Numerical solution of algebraic and transcendental equations; direct and iterative methods for system of linear equations; matrix eigenvalue problems; interpolation and approximations; numerical differentiation and integration; composite numerical integration; double numerical integration; numerical solution for initial value problems; finite difference and finite element methods for boundary value problems.

  - **Probability and Statistics:** Axiomatic approach of probability; random variables; expectation, moments generating functions, density and distribution functions; conditional expectation.

  - **Linear Programming:** Linear programming problem and its formulation; graphical method, simplex method; artificial starting solution; sensitivity analysis;

duality and post-optimality analysis.

- This is only a sample paper and only meant to be indicative of the type of questions that will be asked.

1. If $f'(x)$ and $g'(x)$ exist and $f'(x) > g'(x)$ for all real *x*, then the graph of *y*=*f*(*x*) and *y*=*g*(*x*)

   a. intersect exactly once
   b. intersect no more than once
   c. do not intersect
   d. have a common tangent at each point of intersection

2. If a function *f* is continuous for all real *x* and if *f* has a relative maximum 4 at *x*=-1 and a relative minimum *-2* at *x*=3 then which of the following statements must be true?

   a. $f'(-1) = 0$
   b. The graph of *f* has a horizontal asymptote
   c. The graph of *f* has a horizontal tangent line at x = 3
   d. The graph of *f* intersects both axes

3. If $\frac{1}{u} = \sqrt{x^2 + y^2 + z^2}$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$ is equal to
   a. *u*
   b. −*u*
   c. 0
   d. 1/2

4. If $f(x) = x^3 + 3x^2 + 4x + 5$ and $g(x) = 5$, then $g(f(x))$ is

   a. $5x^3 + 15x^2 + 20x + 25$
   b. 1125
   c. 225
   d. 5

5. Which one of the following equations has a graph that is symmetric with respect to the origin?
   a. $y = \frac{x+1}{x}$
   b. $y = -x^5 + 3x$
   c. $y = x^4 - 2x^2 + 6$
   d. $y = (x-1)^3 + 1$

6. If f is a continuous function on $[a,b]$, which one of the following is necessarily true?

   a. f' exists on (a,b).
   b. The graph of f' is a straight line.
   c. $\lim_{x\to x_0} f(x) = f\left(\lim_{x\to x_0} x\right)$ for $x_0 \in (a,b)$
   d. $f'(x) = 0$ for some $x \in [a,b]$

7. Consider $L = \left(\frac{z}{\underline{z}}\right)^2$. The value of L is

   a. 1 if $z \to 0$ along real axis
   b. -1 if $z \to 0$ along imaginary axis
   c. 1 if $z \to 0$ along line $y = x$
   d. none of these.

8. The function $f(z) = 2x^2 + y + i(y^2 - x)$ is

   a. analytic at one point.
   b. analytic at two points.
   c. nowhere analytic.
   d. none of the above.

9. The derivative of the principal value z^i at the point $z = 1 + i$ is

   a. $\frac{1+i}{2} e^{-\pi/4 + i(ln2)/2}$
   b. $\frac{1-i}{2} e^{-\pi/4 + i(ln2)/2}$
   c. $\frac{1+i}{2} e^{-\pi/4 - i(ln2)/2}$
   d. $\frac{1-i}{2} e^{-\pi/4 - i(ln2)/2}$

10. The integral $\int_C (x^2 + iy^2)dz$, where C is the contour starting from (0,0) along $y = x$ to (1,1) and then along line $x = 1$ to (1,2) is

    a. $7/3 - i5/3$.
    b. $-7/3 + i5/3$.

c. $7/3 + i5/3$.

d. $-7/3 - i5/3$.

11. Let $z_1 = \frac{(4n+1)\pi}{2} - i ln(5 + 2\sqrt{6})$ and $z_2 = \frac{(4n+1)\pi}{2} - i ln(5 - 2\sqrt{6})$. The solution of the equation $sin z = 5$ is

a. z_1 only

b. z_2 only

c. both z_1 and z_2

d. either z_1 or z_2.

12. The value of the integral $\oint_C \frac{e^z}{z^4+5z^3} dz$ where C is the circle $|z| = 2$ is

a. $-\frac{17\pi}{125} i$.

b. $\frac{17\pi}{125} i$.

c. $-\frac{17\pi}{123} i$.

d. $\frac{17\pi}{123} i$

13. The value of the integral $\oint_C \frac{1}{(z-1)^2(z-3)} dz$ where contour C is the rectangle defined by $x = 0, x = 4, y = -1$ and $y = 1$ in anticlockwise direction is

a. -2.

b. 2.

c. 1.

d. 0.

14. If A and B are two disjoint sets with cardinality α and β, respectively, then the cardinality of the set $\{f : f : A \rightarrow B\}$ is

a. $\alpha\beta$
b. β^α
c. α^β
d. $\alpha + \beta$

15. Eigenvalues of a real symmetric matrix are always

a. positive only
b. negative only
c. real
d. real or imaginary

16. The dimension of **R** over **Q** is

a. 1

b. 2

c. n

d. infinite

17. Let $V = C^2$, with the standard inner product and define $T: C^2 \to C^2$ by

$$T([z_1 z_2]) = [2z_1 + iz_2(1-i)z_1],$$

then the adjoint of operator T is

$$a. T([z_1 z_2]) = [2z_1 + (1+i)z_2 - iz_1]$$

$$b.\ T([z_1 z_2]) = [-2z_1 + (1+i)z_2 iz_1]$$

c. $T([z_1 z_2]) = [2z_1 + (1-i)z_2 - iz_1]$

d. $T([z_1 z_2]) = [2z_1 + (1+i)z_2 iz_1]$

18. The orthonormal basis of column space of the matrix $[1-110011-1001-1]$ is

a. $\{[\sqrt{2}/2,0,\sqrt{2}/2,0]^T,[0,0,0,1]^T,[-\frac{\sqrt{6}}{6},\frac{\sqrt{6}}{3},-\frac{\sqrt{6}}{6,0]^T}\}$.

b. $\{[\sqrt{2}/2,0,-\sqrt{2}/2,0]^T,[0,0,0,1]^T,[\frac{\sqrt{6}}{6},\frac{\sqrt{6}}{3},-\frac{\sqrt{6}}{6,0]^T}\}$.

c. $\{[-\sqrt{2}/2,0,\sqrt{2}/2,0]^T,[0,0,0,1]^T,[\frac{\sqrt{6}}{6},\frac{\sqrt{6}}{3},-\frac{\sqrt{6}}{6,0]^T}\}$.

d. $\{[\sqrt{2}/2,0,\sqrt{2}/2,0]^T,[0,0,0,1]^T,[\frac{\sqrt{6}}{6},\frac{\sqrt{6}}{3},-\frac{\sqrt{6}}{6,0]^T}\}$.

19. As $t \to \infty$, the solution of the differential equation $\frac{dy}{dt} = y - y^3$ with initial condition $y(0) = 0.1$ will approach to

a. 1

b. -1

c. 0

d. -2

20. The general solution of the differential equation $y'' + 4y = 3\ cos\ cos 2x + 2x^2$ is

a. $y = c_1\ sin\ sin 2x + c_2\ cos\ cos 2x + \frac{3}{4}x\ sin\ sin 2x + \frac{1}{2}x^2 - \frac{1}{4}$

b. $y = c_1\ sin\ sin 2x + c_2\ cos\ cos 2x - \frac{3}{4}x\ sin\ sin 2x + \frac{1}{2}x^2 - \frac{1}{4}$

c. $y = c_1\ sin\ sin 2x + c_2\ cos\ cos 2x + \frac{3}{4}x\ sin\ sin 2x - \frac{1}{2}x^2 - \frac{1}{4}$

d. $y = c_1\ sin\ sin 2x + c_2\ cos\ cos 2x + \frac{3}{4}x\ sin\ sin 2x + \frac{1}{2}x^2 + \frac{1}{4}$

21. The system of equations $\frac{dx_1}{dt} = x_2, \frac{dx_2}{dt} = x_3, \frac{dx_3}{dt} = x_3 - 2tx_2 + 3x_1 - 6$ is equivalent to

a. x'''+ x'' +2tx'-3x-6 =0

b. x'''- x'' +2tx'-3x-6 =0

c. x'''+ x'' +2tx'-3x+6=0

d. x'''+ x'' +2tx'-3x-6 =0.

22. The solution of the heat equation $u_t = 3\, u_{xx}, u(x,0) = x(\pi - x),\ u(0,t) = u(\pi,t) = 0$ is

a. $u(x,t) = \frac{8}{\pi}\sum_{k=1}^{\infty} \quad \frac{1}{(2k-1)^3} e^{-3(2k+1)^2 t} \sin \sin(2k-1)x.$

b. $u(x,t) = \frac{8}{\pi}\sum_{k=1}^{\infty} \quad \frac{1}{(2k+1)^3} e^{-3(2k-1)^2 t} \sin \sin(2k+1)x.$

c. $u(x,t) = \frac{8}{\pi}\sum_{k=1}^{\infty} \quad \frac{1}{(2k-1)^3} e^{-3(2k-1)^2 t} \sin \sin(2k-1)x.$

d. $u(x,t) = \frac{8}{\pi}\sum_{k=1}^{\infty} \quad \frac{1}{(2k+1)^3} e^{-3(2k+1)^2 t} \sin \sin(2k+1)x.$

23. The Lagrange polynomial that passes through three data points is given by

$f(x) = 24L_0(x) + 37L_1(x) + 22L_2(x)$, where $f(15) = 24, f(18) = 37, f(22) = 25$. The value of $L_1(x)$ at $x = 16$ is

a. 4
b. 2
c. ½
d. 1/4

24. The solution of the partial differential equation $u_x - u_y - 2y = 0$,
$u(x, 2x+1) = e^x$ is

a. $u(x,y) = x^2 - 2xy - e^{\frac{x+y-1}{3}} - 6\left(\frac{x+y-1}{3}\right)^2 - 2\left(\frac{x+y-1}{3}\right).$

b. $u(x,y) = x^2 + 2xy + e^{\frac{x+y-1}{3}} + 6\left(\frac{x+y-1}{3}\right)^2 + 2\left(\frac{x+y-1}{3}\right).$

c. $u(x,y) = x^2 + 2xy - e^{\frac{x+y-1}{3}} + 6\left(\frac{x+y-1}{3}\right)^2 - 2\left(\frac{x+y-1}{3}\right).$

d. $u(x,y) = x^2 + 2xy + e^{\frac{x+y-1}{3}} - 6\left(\frac{x+y-1}{3}\right)^2 - 2\left(\frac{x+y-1}{3}\right).$

25. The solution of heat equation $u_{tt} = 5\, u_{xx}, u(x,0) = \sin \sin x\ ,\ u_t(x,0) = \sin \sin 3x$ is

a. $u(x,t) = \frac{1}{2}\left[\sin \sin\left(x + \sqrt{5}t\right) - \sin \sin\left(x - \sqrt{5}t\right)\right] - \frac{1}{30}\left[\cos \cos\left(x + \sqrt{5}t\right) - \cos \cos\left(x - \sqrt{5}t\right)\right]$

b. $u(x,t) = \frac{1}{2}\left[\left(x - \sqrt{5}t\right) - sin\ sin\left(x + \sqrt{5}t\right)\right] - \frac{1}{30}\left[cos\ cos\left(x - \sqrt{5}t\right) - cos\ cos\left(x - \sqrt{5}t\right)\right].$

c. $u(x,t) = \frac{1}{2}\left[sin\ sin\left(x - \sqrt{5}t\right) - sin\ sin\left(x - \sqrt{5}t\right)\right] - \frac{1}{30}\left[cos\ cos\left(x + \sqrt{5}t\right) - cos\ cos\left(x - \sqrt{5}t\right)\right].$

d. $u(x,t) = \frac{1}{2}\left[sin\ sin\left(x - \sqrt{5}t\right) - sin\ sin\left(x - \sqrt{5}t\right)\right] + \frac{1}{30}\left[cos\ cos\left(x - \sqrt{5}t\right) - cos\ cos\left(x - \sqrt{5}t\right)\right].$

26. The solution of the partial differential equation $9\ u_{xx} + 12\ u_{xy} + 4u_{yy} = 0$ is given by

$a.\ u = xf(2x + 3y) + g(2x - 3y).$

$b.\ u = xf(2x - 3y) + g(2x - 3y).$

$c.\ u = xf(2x - 3y) + g(2x + 3y).$

$d.\ u = xf(2x - 3y) - g(2x + 3y).$

27. The Green's function for the BVP $y'' = -f, y(0) = y(\pi) = 0\ ,\ 0 < x < \pi$ is

$a.\ G(x;\xi) = \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{sinsinnxsinsinn\xi}{n^2}.$

$b.\ G(x;\xi) = \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{sinsinnxcoscosn\xi}{n^2}.$

$c.\ G(x;\xi) = \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{coscosnxsinsinn\xi}{n^2}.$

$$d.\ G(x;\xi) = \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{cos\ cosnx\ cos\ cosn\xi}{n^2}$$

28. The hospital period, in days, for patients following treatment for a certain type of kidney disorder is a random variable $Y = X + 4$, where X has the density function

$$f(x) = \frac{32}{(x+4)^3}, \qquad x > 0{,}0, \qquad otherwise.$$

Find the probability that the hospital period for a patient following this treatment will exceed 8 days.

a. $\frac{1}{4}$

b. $\frac{1}{5}$

c. $\frac{2}{9}$

d. $\frac{3}{4}$

29. Consider two independent random variables $X$and Z with means 3 and 4, and variances 9 and 16, respectively. For a random variable $Y = \frac{X}{Z,}$ the variance of Y is approximately equal to

a. $\frac{140}{9}$

b. 10

c. $\frac{153}{16}$

d. 9

30. Let$f$be a function defined by $f(x) = \int_1^x \quad t(t^2 - 3t + 2)dt, \quad x \in [1,3]$. If a number is randomly selected from the domain of f, then the chance that the selected number also belongs to range of f is

a. $\frac{1}{6}$

b. $\frac{1}{2}$

c. $\frac{1}{4}$

d. $\frac{1}{8}$

31. If $xy = a^2$ and $S = b^2x + c^2y$, where a, b and c are randomly selected from intervals[0,1], [2,4] and [6,9], respectively, then the probability of minimum value of S lying in the interval [0,72] is

a. 0

b. $\frac{1}{2}$

c. 1

d. $\frac{2}{3}$

32. Suppose that the two-dimensional random variable (X, Y) is uniformly distributed over the region

$$R = \{0 < x < y < 1\}.$$

The correlation coefficient between X and Y is given by

a. $\frac{1}{2}$

b. $\frac{1}{3}$

c. $\frac{2}{3}$

d. $\frac{1}{4.}$

33. Which of the following is unique factorization domain (UFD) but not principal ideal domain (PID)

a. Z[X]
b. Q[x]
c. Z
d. R[x].

34. If G is a group of order 175, then which of the following statements is false?

a. G is abelian

b. G is cyclic

c. G is simple

d. G has a Sylow 5 subgroup of order 25

35. Which of the following groups is simple?

a. A group of order 108

b. A group of order 148

c. A group of order 56

d. A group of order 17

36. The Order of 5 + <6>in $\mathbf{Z}_{18}$\ <6>is

a. 1

b. 2

c. 3

d. 6

37. Given $\frac{d^2y}{dx^2} = 6x - \frac{1}{2}x^2, y(0) = 0, y(12) = 0$, the value of $\frac{d^2y}{dx^2}$ at $x = 4$ using the finite difference method and a step size $h = 4$ can be approximated by

a. $\frac{y(0)+y(8)-2y(4)}{16}$

b. $\frac{y(0)-y(8)-2y(4)}{16}$

c. $\frac{y(0)+y(8)+2y(4)}{16}$

d. $\frac{y(0)+y(8)-2y(4).}{4}$

38. The sufficient condition for Jacobi iterative method $X^{(k+1)} = BX^{(k)} + C, k = 0,1,2 \dots \dots$ for solving the system of linear equations is given by

a. $\|B\| < 1/2$

b. $\|B\| < 3$

c. $\|B\| < 1$

d. $\|B\| > 1$.

39. The approximated value of $y(1.1)$ for the initial value problem $\frac{dy}{dx} + 2xy^2 = 0,\ y(1) = 1$ by Euler method using step size $h = 0.1$ is given by

a. 0.1

b. 0.7

c. 0.9

d. 0.8

40. If Δ and ∇ denote forward and backward difference operators, then which of the following is true?

a. $\Delta + \nabla = \frac{\Delta}{\nabla} + \frac{\nabla}{\Delta}$

b. $\Delta + \nabla = \frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}$

c. $\Delta + \nabla = -\frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}$

d. $\Delta + \nabla = \frac{\Delta}{\nabla} / \frac{\nabla}{\Delta}$

41. The boundary value problem $\frac{d^2y}{dx^2} = y^2, y(0) = 1, y(1) = 2$, is approximated by the difference equation $y_0 = 1, y_{n+1} = 2, y_{k-1} - 2y_k + y_{k+1} = h^2 y_k^2, k = 1(1)n$. The order of the difference equation is

a. 1

b. 2

c. 3

d. 4

42. Picard's method of successive integration fails if

a. the function is not measurable.

b. the function is not monotone.

c. the function is not bounded.

d. the function is not integrable.

43. Using Simpson's 1/3 rd rule of integration with step size , the value of integral correct upto three decimal places is

a. 0.695

b. 0.691

c. 0.694

d. 0.693

44. A variable which does not appear in the basis variable column of simplex table is

a. Never zero
b. Always equals to zero
c. Called basic variable
d. None of these.

45. Suppose that the cost of performing an experiment is 1000 rupees. If the experiment fails, an additional cost of 300 rupees occurs because of certain changes that have to be made before the next trial is attempted. If the probability of success on any given trial is 0.2, if the individual trials are independent, and if the experiments are continued until the first successful result is achieved, what is the expected cost of the entire procedure?

a. 6500 Rs

b. 6200 Rs

c. 5600 Rs

d. 6000 Rs

46. The following four inequalities define a feasible region. Which one of these could be removed from the list without changing the region?

a. $x-2y \geq 8$
b. $y \geq 0$
c. $-x-y \leq 10$
d. $x+ y \leq 20$.

47. If an artificial variable is present in the basic variable column of an optimal simplex table, then the solution is

a. unbounded
b. infeasible
c. optimal

d. None of these

48. A solved LP problem indicated that the optimal solution was $x_1 = 10$ and $x_2 = 20$. One of the constraints was $4\ x_1 + 2\ x_2 \leq\ 80$. This constraint has

a. surplus greater than zero.

b. slack greater than zero.

c. surplus equal to zero.

d. slack equal to zero.

49. If in a standard form LPP, we have a linear system with 20 nonnegative variables and 10 equations, then the maximum number of basic solutions is given by

a. 200

b. 2

c. 184756

d. 670442572800

50. Which of the following is not a feasible solution of the dual of the given problem:

$Max x_1\ +\ 2x_2$ subject to

$$x_1\ +\ x_2\ \leq\ 4$$

$$-x_1\ +\ x_2\ \leq\ 1$$

$$2x_1 - x_2 \leq\ 3$$

$$x_1, x_2\ \geq\ 0$$

a. (0, 1/2, 1/2)

b. (2, 1, 1)

c. (3/2, 1/2, 0)

d. (2, 1, 2)